An Examination of Learning Using Fourier Analysis of Mathematical Models of Consciousness

Mary Foss
Weber State University, USA

Yucheng Liu
South Dakota State University, USA

Shantia Yarahmadian
Mississippi State University, USA

www.ijonest.net

To cite this article:
Foss, M., Liu, Y., & Yarahmadian, S. (2024). An examination of learning using Fourier analysis of mathematical models of consciousness. *International Journal on Engineering, Science, and Technology (IJonEST)*, *6*(3), 236-250. https://doi.org/10.46328/ijonest.209

International Journal on Engineering, Science, and Technology
2024, Vol. 6, No. 3, 236-250 https://doi.org/10.46328/ijonest.209

An Examination of Learning Using Fourier Analysis of Mathematical Models of Consciousness

Mary Foss, Yucheng Liu, Shantia Yarahmadian

Article Info

Article History

Received:
29 November 2023
Accepted:
20 May 2024

Keywords

Consciousness
Mathematical model of learning
Fourier transform
Project-based learning

Abstract

The concept of consciousness remains quite possibly the ultimate mystery for humanity. It is somehow linked to the ability to learn and experience the external environment. The process of learning is of utmost importance to any species and yet remains largely not understood. While many experts from varying fields throughout history have attempted to quantify and measure learning, all failed to capture the process with a mathematical explanation. The purpose of this paper is to present three different models of consciousness and their related Fourier transform and evaluate the ability of each to capture some of the behavior that is understood about the conscious experience. There is some evidence that the experience of consciousness as a function of time can be measured in an expression of phase. With this hypothesis, the technique of Fourier transform becomes a useful tool to examine a mathematical model of consciousness that can be transformed from the phase plane to the linearization of time. This paper summarizes some current research on consciousness and learning and perception of time. This paper then presents three different models of consciousness represented as a pulse function, Dirac Delta function, and exponential decay function and examines each model utilizing Fourier transform. Finally, this paper concludes that the intersection between psychology, neuroscience and cognitive science can be made by utilizing analysis tools in the field of applied mathematics.

Introduction

Borrowing from the musings of Galileo, it can be said that mathematics is the language of nature and as such can be a way to understand certain phenomena that remain a mystery and yet will continue to inspire human thought. The question of what it is to exist has so famously been asked by not only artists and philosophers but likely every human that has ever walked this earth. While this question will likely remain a mystery and indeed is perhaps a driving force of the very nature of human curiosity, one might endeavor to understand the question more by looking to nature and referring to the tools available in the modern world. One such mystery is the process of learning and how it relates to theories of consciousness. Learning can be thought of as a form of consciousness (Tyler, 2020). Though, in general, the concept of consciousness is controversial and remains undefined (Gamez, 2014), learning can occur at conscious as well as unconscious states (Seth, 2008) and is largely agreed to be related to time. The distinction devolves on the conception of time. When considering the role of time in learning, time

has been spatialized as clock time. However, when considering thermodynamics, for example, time is a measure of process relating to how quickly energy is expended. Brain processes likely are modeled based on both aspects of time as an external measurement and internal process

Background

Mathematical Models of Consciousness

Understanding the conscious experience and its relation to the physical domain has been the interest of philosophers, theologians and scientists for hundreds of years. As the field of neuroscience has expanded in the past several decades alongside cognitive psychology and analytical philosophy, the science of consciousness as a field of study has been firmly created. The aim of this research is to address the relationship between consciousness and the physical domain through Global Workspace Theories, Multiple Draft Theory, Higher Order Thought Theories or Integrated Information Theory among others.

Other theories that are mathematical in nature are also rising in importance. Examples include Integrated Information Theory (IIT) and Predictive Processing Theory (Scheider, 2017). However, there continue to exist numerous challenges in the science of consciousness; namely, it is a phenomenon unlike any other studied by natural science in that is a purely subjective experience. There also exist fundamental aspects of consciousness that are not readily available and contained in the unconscious or subconscious mind (Kleiner, 2020).

Existing models take different approaches in the philosophy of the conscious experience. However, many are built upon the work of David Chalmers, who defined the physical domain as including materials (neurons, brain tissue, atoms, etc.) as well as physical notions such as mass and space-time. He assumes that the physical domain is causally closed and for every physical event there is a physical sufficient cause (Chalmers, 2010, 1996). Chalmers also assumes aspects of consciousness are not associated with a function or structure and therefore does not have a causal influence on the physical domain. Kleiner's work focuses on developing a general mathematical framework for models of consciousness that can be employed in the process of developing the theory while evaluating existing models that have been developed (Kleiner, 2020).

Namazi and Kulish introduced a mathematical based definition of human consciousness based on macroscopic brain organization that has been observed in global brain electroencephalography (EEG) signals. They defined consciousness as an energy transmission between the physical phase space and the mind phase space through energy flux in the form of sensation or action and developed a partial differential equation (PDE) to describe the conservation of energy. The PDE is then solved with a convolution integral resulting in a fluctuation function that is represented as a series of Gaussian pulses (Namazi, 2012).

The goal of this paper is to link the understood wave-like behavior of brain activities or energy motions to the linearization of time. This is done by first developing a time-based model of consciousness and then examining the resulting Fourier transform.

Projective Consciousness Model (PCM)

Rudrauf et al. introduced a mathematical model of consciousness (projective consciousness model) that hypothesizes that the spatial field of consciousness is a projective geometry as shown in Eq. 1 (Rudrauf, 2017).

$$S(X,t) = T(t) \circ R(X,t) \tag{1}$$

Where R(X, t) defines the state at time t of a set of X internal variables that describe the external world and T(t) is the process of projective transformation which allows for the subjective experience of space and our environment at a given instant S(X, t). The symbol, ∘, corresponds to the application of the projective transformation, T, to the spatial world model, R (Rudrauf, 2017).

The goal of PCM is to contribute to the overarching goal of mathematizing of the conscious phenomena. It does so in an incremental way to allow for further development of a model and that can connect with other fields. One goal of the present study is to present a hypothesis that the projective transformation aspect of this model is in fact the Fourier transform operator and the subjective experience of the individual S(X, t) should instead be represented as S(X, θ).

Consciousness in the Phase Plane

The concept of consciousness existing in the phase plane has been studied before. Joye proposed that the integration of Karl Pribham's Holonomic Brian theory and David Bohm's interpretation quantum theory leads one to hypothesize that the range of human consciousness is a function of the mind in the frequency domain. Joye challenged the assumption that consciousness is generated through the action of neuronal spikes in the brain and suggested that the examination of electromagnetic field radiation is a possible basis for consciousness (Joye, 2017; Pribram, 1986).There is not consensus on the definition of consciousness, much less what it is called and how it could be modeled. The goal of this study is not to address this topic but rather form a model based on the premise that there are some cyclical behaviors that is expected in a model of consciousness.

Learning Schema

One way to address the subjective elements of consciousness present in all life forms is to instead focus on the relationship of consciousness to learning. While the definition of consciousness remains somewhat ambiguous, Ordahl concludes that in learning, conscious and unconscious factors exist and occurs without necessarily awareness of that fact that one is learning (Ordahl, 1911). The Greek philosopher Plato is credited with the proverb: All learning is remembering and based upon this described the Socratic method of instruction (Schneider, 2013). Humanity has been fascinated by the process of learning since the beginning of written records. In reality, we must attribute the whole of modern civilization to this mysterious process. Through learning, we are able to build upon knowledge and that knowledge can be disseminated and shared for others to learn and also build upon. Illeris describes the fundamental process of learning with three basic labels shown in Figure 1 (Illeris, 2018).

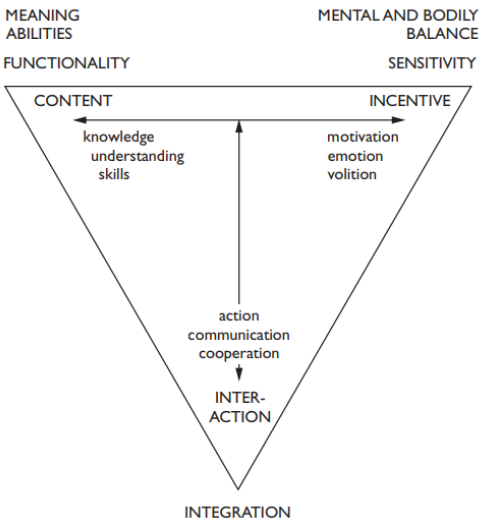

Figure 1. Illeris' Three Dimensions of Learning (Illeris, 2018).

From these 3 schemas, a multitude of learning theories have been developed, tested, and studied with all some degree of success. While it is outside the scope of this paper to discuss how they are dissimilar, it is relevant to discuss in what ways the theories overlap. In general, each learning theory discusses learning as a process that takes place in stages and in cycles which creates a disconnect between empirical data in the time domain and allows for the hypothesis that the way that learning occurs happens in the frequency domain. In general, the process of learning is not well understood but is of utmost importance. Efforts to determine the feasibility of understanding behavior such as stability through the tools of applied mathematics are useful in so far that certain layers of human cognition might be illuminated and potentially lead to a mathematical model of learning that was deterministic instead of stochastic. Foss and colleagues have studied the application of learning through project-based learning (Foss et al, 2020, 2021, 2022) and examined the feasibility of creating a mathematical model of the learning process based upon reaction kinetics (Foss et. al, 2022).

There is significant evidence that there is a disconnect between the measurement of time and the experience of moving through time which somehow relates to consciousness. This is a topic that is of interest to many fields in neuroscience and psychology. There is ample research that describes the subjective nature of time (LiDernia, 2018; Malapani, 2002; Fayolle, 2015; Droit-Volet, 2017, 2016; Wearden, 2015) and how that changes depending on circumstances (Droit-Volet, 2015, Wearden, 2013). The experience of time is influenced by age, emotion, as well artificially by drugs such as d-amphetamine (Droit-Volet, 2015, Wearden, 2013; Sarigiannidis, 2020; Lake, 2013; Lucas, 2014). As a result, there remains a disconnect in understanding any conscious process, such as learning, using time as a measured variable.

The matter is possibly understood deeper when examining the history of time as a concept. As such, the ability to measure time with any degree of accuracy is a comparatively modern phenomenon that has only become widely available with the availability of accurate clocks that was first invented in 1888. Prior to this, representing the bulk of human history, time would have been measured as a cycle. Time was measured by the seasons, the position of the sun in the sky, the stars and other cues that have always been available leading to the hypothesis that instead

of considering time as a fixed variable, perhaps instead we experience life disconnected with the elapsing time and instead as a cycle or in other words, a frequency.

One such tool to examine the relationship between the linear nature of measured time involved in learning and the apparent cyclical nature in which learning is accomplished is the Fourier transform. While the mystery of consciousness is probably the most commonly pondered topic, there are certain elements that are understood about what it means to be conscious. In this study, we evaluate three different models of consciousness to analyze what elements can be captured.

Fourier Transform

Fourier transform is an integral operator that functions to change a space-time coordinate system into a spectral coordinate system and has many applications in engineering and physics, such as image processing, vibration and acoustic analysis, wave energy analysis, and materials modeling (Logan, 2003). Eqn. (2) defines the Fourier transform, where the variable, ξ represents the wave number variable.

$$(\mathcal{F}u)(\xi) = \hat{u}(\xi) = \int_{-\infty}^{\infty} u(x)e^{i\xi x}\,dx \tag{2}$$

For this study, the wave number variable will be substituted for ξ and the analysis will be performed with equation 3.

$$(\mathcal{F}u)(\omega) = \hat{u}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} u(x)e^{i\omega x}\,dx \tag{3}$$

The importance of Fourier transform in the field of applied mathematics cannot be overstated. The integral operator is now the base of signal processing, imaging, probability theory, as well as solving differential equations (Logan, 2003).

Philosophical Basis of Model

This paper presents three different mathematical models that capture the experience of consciousness and the resulting Fourier transform. The goal of the present study is to not presume that any of the models are correct but instead to evaluate the ability of each model to capture certain elements that are qualitatively understood about the conscious experience rather than what can be quantified. There are many phenomena related to the conscious experience that cannot be measured nor understood. For example, an individual may refer to feeling down or instead feeling very upbeat.

Mathematical language is used to describe the experience of being consciousness not to quantify this experience but rather capture elements of behavior. This behavior is imagined as an internal amplitude of an individual and our goal is to represent behavior that is unquantifiable. As a result, numerical values presented in the graphs are not intended to represent measurements.

Consciousness as a Delta Function

Perhaps the simplest mathematic model to describe consciousness is the impulse function known as the Dirac Delta Function as shown in Eqn. 1. In this model, at a certain time, life begins to exist and at some time later, life ceases to exist. When using the assumption that the universe has existed in the negative infinite direction as well as positive infinite direction, it is reasonable that the duration of an entire sentient life is infinitesimally small. Our conscious awareness of existing in any other state or form is not subject to either prove or disprove as is the possibility of determining a value of consciousness. As such, there are certain realities that are captured by using the delta function as a model for consciousness – an experience that is immeasurable and is non-function. At a point x_0 consciousness exists and exists in an infinite way. The duration of consciousness is infinitesimally small. By utilizing the Fourier Transform, more insight into this function can be obtained.

At $x=x_o$, the Dirac Delta function has an infinite value but remains zero everywhere else, making this a non-function. As such, this function is an example of a distribution which is defined in terms of its integration properties. Despite this, many natural phenomena can be described by the behavior of this distribution as it relates to modeling sudden shocks or large forces to a system.

$$\int_{-\infty}^{+\infty} f(x - x_o)\, \delta(x) dx \tag{4}$$

The Fourier transform of the Dirac distribution is then calculated for the position at x = 0.

$$F(\omega) = \mathcal{F}\{\delta(x)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\omega x}\, \delta(x) dx = \frac{1}{2\pi} \tag{5}$$

As shown in Figure 2, the perfect spike of the delta function concentrated at x = 0 is transformed into a constant function in the frequency domain.

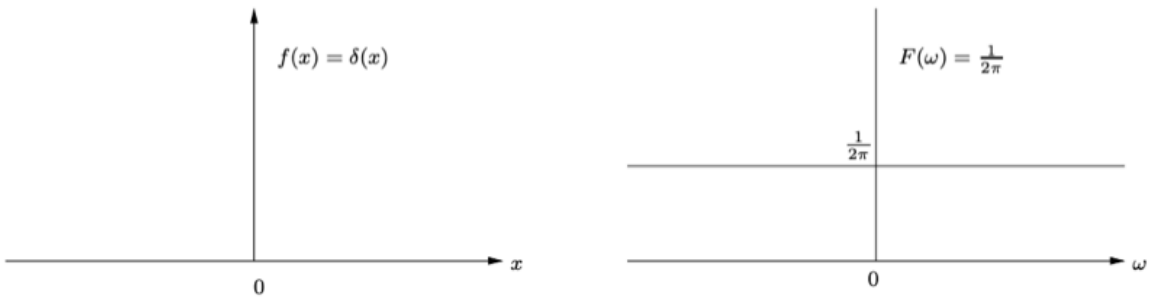

Figure 2: Dirac Delta Function and Associated Fourier Transform.

Assuming it is logical that consciousness is shifted from x = 0 to some value $x = x_0$, the Fourier Transform becomes the following:

$$F(\omega) = \mathcal{F}\{\delta(x - x_0)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-i\omega x}\,\delta(x - x_0)dx = \frac{1}{\sqrt{2\pi}}e^{i\omega x_0} \quad (6)$$

Amazingly, Eqn. (6) results in Euler's Equation shown in Eqn. (7), which is often cited as an example of deep mathematical beauty and is almost unanimously regarded as one of the most truly mysterious and remarkable discoveries as it relates a deep relationship between the complex exponential function and trigonometry. Eqn. (6) is rewritten in the form in Eqn. (8).

$$e^{ix} = \cos x + i \sin x \quad (7)$$

$$\frac{1}{\sqrt{2\pi}}e^{i\omega x_0} = \frac{1}{\sqrt{2\pi}}\left[\cos \omega x_0 + i \sin \omega x_0\right] \quad (8)$$

This result allows for a possible mathematical interpretation of the manner in which we perceive consciousness as it relates to learning as shown in Fig. 3. There are additional facets of what is intuitively understood about consciousness that are represented by this model:

1. An individual's consciousness is dependent upon the time shift.

a. The conscious experience is dependent upon the era that you are alive

2. There is a real and imaginary or complex component of consciousness,

a. This is perhaps representing the existence of the conscious and unconscious states.

3. There is a cyclical nature in which life is experienced in the frequency domain that can describe the subjective sense of time.

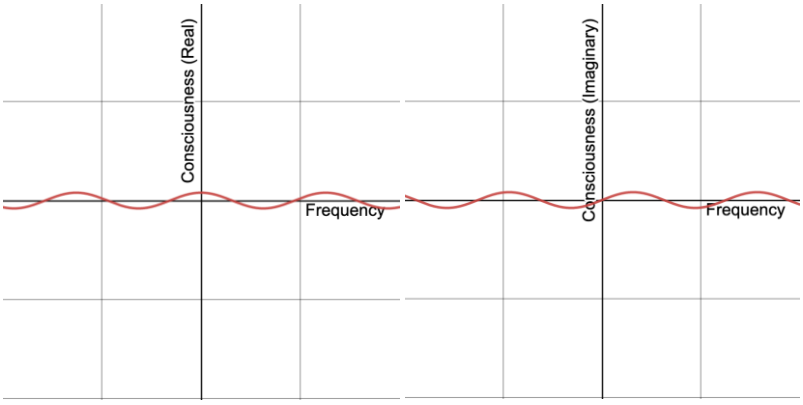

Figure 3: Graph of e^{ix} in; (1) Real Space; (2) Imaginary Space.

Consciousness as a Pulse Function

Eliminating the assumption that consciousness is infinitesimally small (represented as the Dirac Delta function) results in the creation of a separate mathematical model of consciousness. The pulse function shown in Figure 4 is one possible model for this behavior. In this model, consciousness is assumed to be uniform and exist for a

certain duration of measured time. Modeling consciousness as a pulse function demonstrates the mysterious relationship with living and measured time. In this model as time elapses life continues and with it, consciousness.

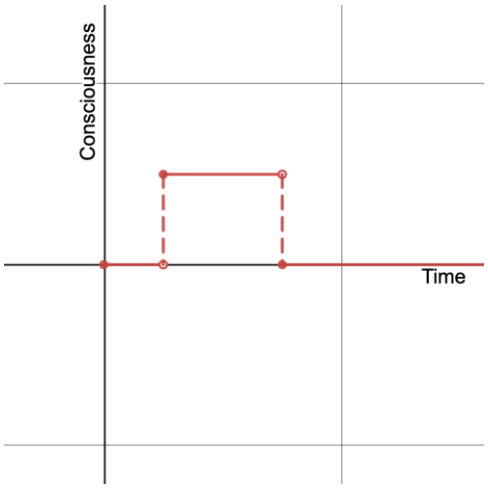

Figure 4. Consciousness as a Pulse Function.

Applying the Fourier transform to the pulse function as shown in Eqn. (9) illuminates and captures some very interesting phenomenon. The following will explore the possibility of representing life as a function of frequency instead of time and what observations can be made. It is important to note that there is no significant meaning to t = 0 other than it represents the midpoint of the pulse function.

$$P_d(t) = \begin{cases} -1, t < 0 \\ 0, t \geq 0 \end{cases} \quad \text{Pulse Function} \tag{9}$$

$$F(\omega) = \mathcal{F}\{P_d(t)\} = \frac{1}{\sqrt{2\pi}} \int_{-d}^{+d} e^{-i\omega t}\, dt = \frac{1}{\sqrt{2\pi}} \frac{1}{i\omega} e^{i\omega t} \Big|_{-d}^{d} \tag{10}$$

$$F(\omega) = \sqrt{\frac{2}{\pi}} \frac{\sin \omega d}{w} = \sqrt{\frac{2}{\pi}} d\,sinc(\omega d) \tag{11}$$

What is interesting about the result of this function is the lifespan or duration variable, d, affects the behavior of the frequency response. By examining this result further, it is possible to compare how consciousness may compare in terms of the amplitude or cyclical nature of frequency as related to the duration of life. Figure 5 represents the Fourier transform of the pulse function when the constant d is equal to 0.1. Recalling that d represents the width of the pulse function, this variable becomes analogous to the potential or average lifespan of a particular creature. As can be seen in this graph, the frequency range is quite uniform.

This would lead to the conclusion that a creature with a very short life span would experience life not as a cycle but as a uniform experience in perception. As there is little research on the perception of time for an insect such as a cicada or other short-lived creature, perhaps this does illuminate a certain level of consciousness that is available to the creatures as they are experiencing their lives. A short-lived creature is not aware that it has a short life and perhaps as a result does not experience the cyclical nature of life.

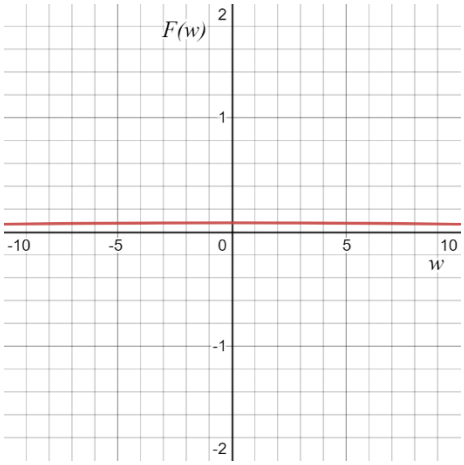

Figure 5: Fourier Transform of Pulse Function, d = 0.1.

In Figure 6, the variable d is increased to 0.5. As can be seen from that figure, there are now a variety of frequencies available as the life is occurring. This could be applied to a creature with a lifespan that is 5 times longer than a cicada but remains still rather short compared to the human lifespan. Interestingly, the frequency range is bounded but does offer a range of lower frequencies with all having a positive amplitude.

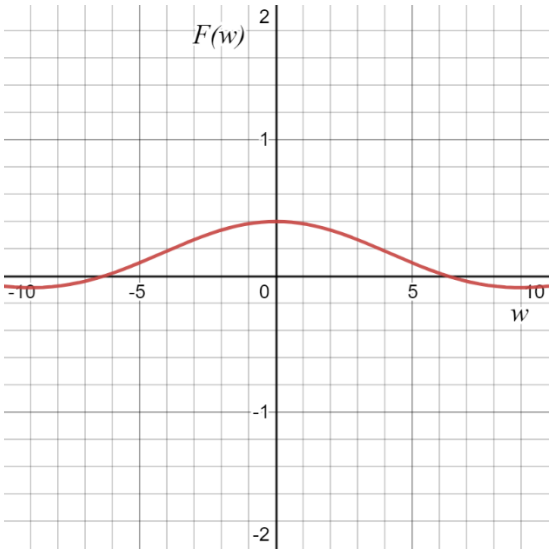

Figure 6: Fourier Transform of Pulse Function, d= 0.5

As the variable d is increased to 10 as shown in Figure 7 and to 50 as shown in Figure 8, the graph of life as a frequency becomes much more cyclical in nature. By utilizing the Fourier transform, one can see that the existence of life modeled by a pulse function becomes a very interesting function in terms of frequency. The amplitude of frequency is determined by the duration of the life or constant d. Additionally, as the duration of the life increases, the amount of variation within the frequency increases, indicating that a living creature would have a greater opportunity to experience a wide range of frequencies throughout their existence with varying amplitudes. At this stage, it is worth exploring the concept of the experience of life at varying amplitudes in both positive and negative directions. This could explain a common perception of consciousness that is widely experienced when an individual may describe the subjective sensation of feeling negative versus positive.

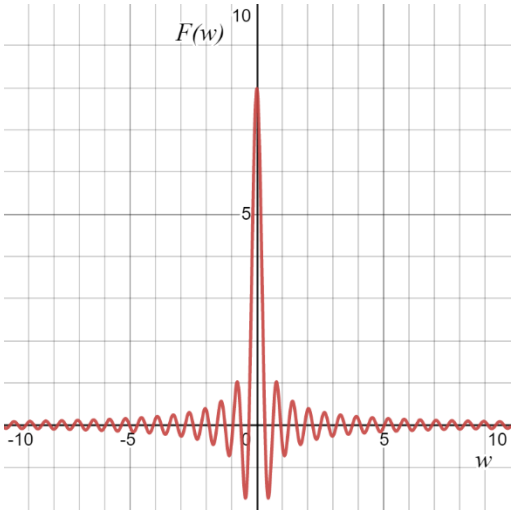

Figure 7: Fourier Transform of Pulse Function, d = 10.

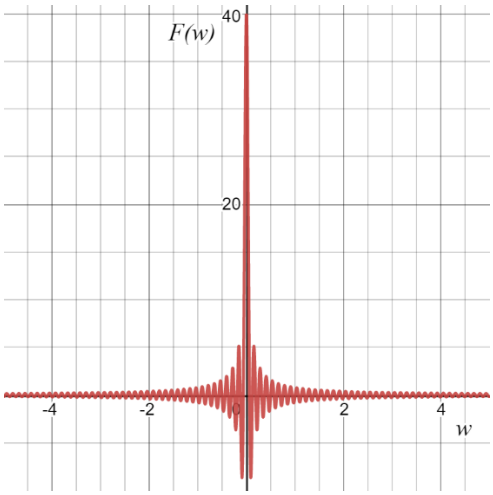

Figure 8: Fourier Transform of the Pulse Function, d = 50

There are additional facets of what is intuitively understood about consciousness that are represented by this model:

1. An individual's consciousness is dependent upon lifespan.
2. There is a cyclical nature in which life is experienced in the frequency domain that can describe the subjective sense of time.

Consciousness as an Exponential Decay Function

Another hypothesis of consciousness as it relates to time is to represent it as a function of exponential decay as shown in Fig. 9. There are some facets of intuitive understanding of the learning process that are captured in this shape. In particular, it has been shown that the brains of babies and children are exceedingly better at learning than those of adults (Droit-Volet, 2017) and also experience time in a different manner than adults (Lucas, 2014).

$$f(t) = \begin{cases} e^{-at} & t > 0 \\ 0 & t < 0 \end{cases} \qquad \text{a} > 0 \tag{12}$$

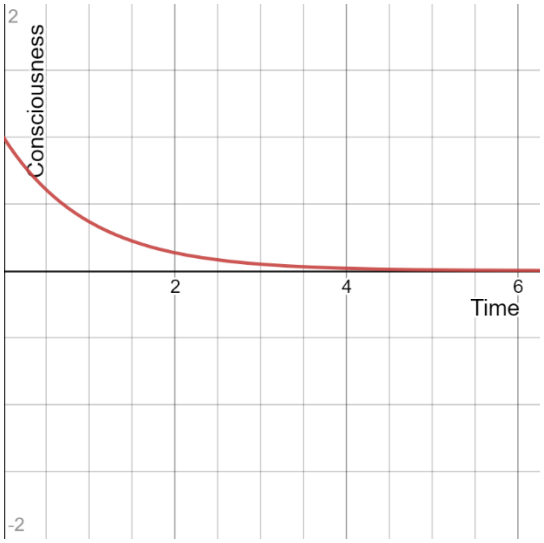

Figure 9: Exponential Decay Function.

$$F(\omega) = \mathcal{F}\{f(t)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t} dt = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-at}\, e^{-i\omega t} dt \tag{13}$$

$$\frac{1}{\sqrt{2\pi}}\frac{-1}{\text{a}+\text{i}\omega} e^{-at} e^{-i\omega t} = \frac{1}{\sqrt{2\pi}(\text{a}+\text{i}\omega)} \tag{14}$$

To better understand the result of this transformation the norm and the phase can be calculated by converting the solution to the form a + *i*b as shown below.

$$F(\omega) = \frac{1}{\sqrt{2\pi}}\left[\frac{a}{a^2+\omega^2} - i\left[\frac{\omega}{a^2+\omega^2}\right]\right] \tag{15}$$

$$\text{r} = [z] = \sqrt{\frac{a^2}{(a^2+\omega^2)^2} + \frac{\omega^2}{(a^2+\omega^2)^2}} = \frac{1}{\sqrt{a^2+\omega^2}} \tag{16}$$

$$\theta = tan^{-1}\frac{\omega}{a} \tag{17}$$

Figures 10 and 11 represent the graphical representation of the phase and amplitude shown in Eqns. (16)-(17) with a = 0.5.

There are additional facets of what is intuitively understood about consciousness that are represented by this model:

1. An individual's consciousness is dependent upon the rate of exponential decay
2. There is a real and imaginary or complex component of consciousness
3. This is perhaps representing the existence of the conscious and unconscious states.

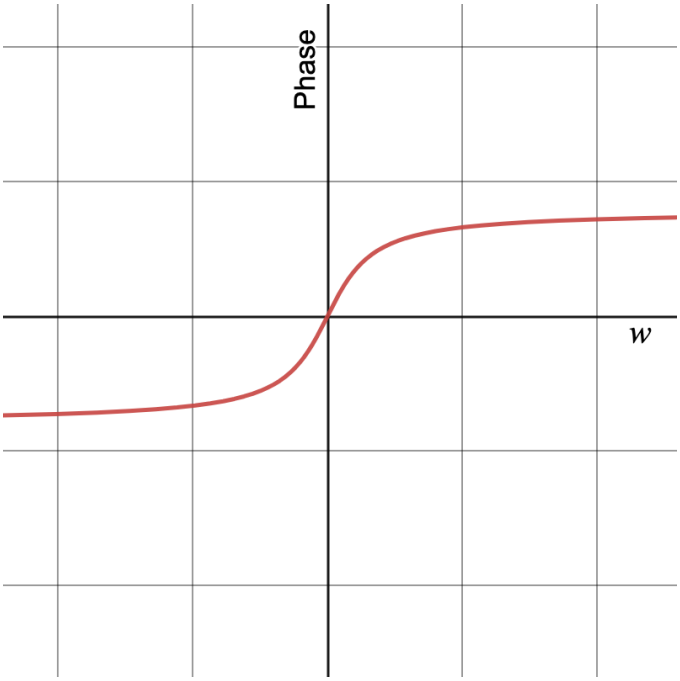

Figure 10: Phase Diagram of Exponential Decay Function Transform with a = 0.5.

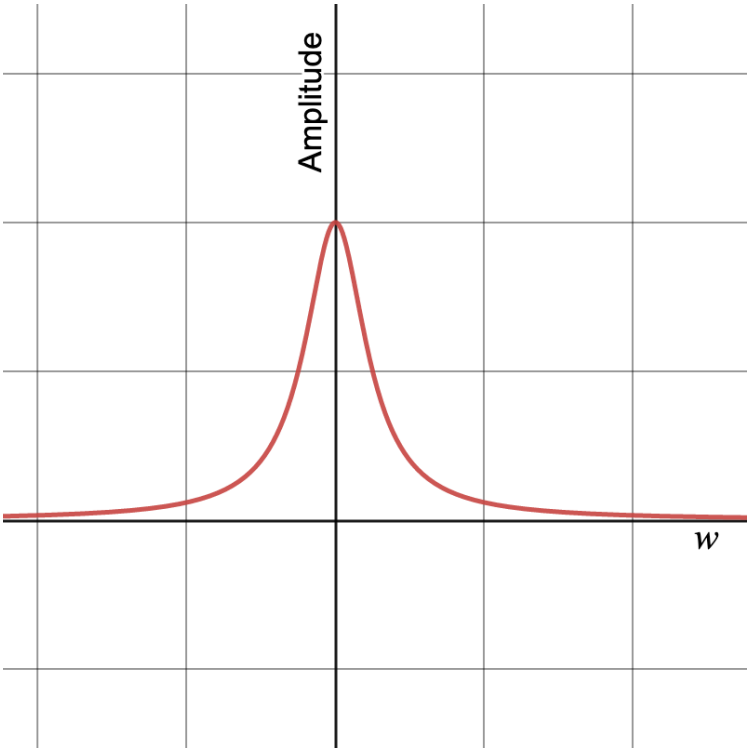

Figure 11: Amplitude of Exponential Decay Function Transform with a = 0.5

Conclusion

Galileo said, “Natural philosophy is written in this grand book the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and to read the alphabet in which it is composed. It is written in the language of mathematics, and its characters are triangles,

circles, and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth." If there was a way to model the complex process of learning, it would undoubtedly be written in the language of mathematics and likely would have parallels to other natural phenomena.

The concept of consciousness remains quite possibly the ultimate mystery for humanity. It is somehow linked to the ability to learn and experience the external environment. The process of learning is of utmost importance to any species and yet remains largely not understood and has behavioral features that are cyclic. While many experts from varying fields throughout history have attempted to quantify and measure learning, all fail to capture the process with a mathematical explanation. There is some evidence that the experience of consciousness as a function of time can be measured in an expression of phase. With this hypothesis, the technique of Fourier transform becomes a useful tool to examine a mathematical model of consciousness that can be transformed from the time plane to the phase plane. This paper summarizes some current research on consciousness and learning and perception of time that supports this hypothesis.

In this paper, we present three different models of consciousness, and their related Fourier transforms and evaluate the ability of each to capture some of the behavior that is understood about the conscious experience. Each of the presented models have limitations but each can elucidate certain aspects of the conscious experience. These models include a pulse function, Dirac Delta function, and exponential decay function and each model is examined utilizing Fourier Transform. The analysis results demonstrate that the intersection between psychology, neuroscience and cognitive science can be made by utilizing analysis tools in the field of applied mathematics.

References

Chalmers, D. (1996). *The Conscious Mind: In Search of a Fundamental Theory*. Oxford University Press, New York.

Chalmers, D. (2010). *The Character of Consciousness*. Philosophy of Mind. Oxford University Press, New York and Oxford.

Di Lernia, D., Serino, S., Pezzulo, G., Pedroli, E., Cipresso, P., & Riva, G. (2018). Feel the Time. Time Perception as a Function of Interoceptive Processing. *Frontiers in Human Neuroscience, 12*. 74. https://www.frontiersin.org/article/10.3389/fnhum.2018.00074

Droit-Volet, S. (2017). Time dilation in children and adults: The idea of a slower internal clock in young children tested with different click frequencies. *Behavioural Processes, 138*, 152-159, ISSN 0376-6357, https://doi.org/10.1016/j.beproc.2017.03.005.

Droit-Volet, S. & Coull, J. (2016). Distinct developmental trajectories for explicit and implicit timing. *Journal of Experimental Child Psychology, 150*, 141-154. https://doi.org/10.1016/j.jecp.2016.05.010.

Droit-Volet, S., Lamotte, M., & Izaute, M. (2015). The conscious awareness of time distortions regulates the effect of emotion on the perception of time. *Consciousness and Cognition, 38*, 155-164, ISSN 1053-8100, https://doi.org/10.1016/j.concog.2015.02.021.

Fayolle, S., Gil, S., & Droit-Volet, S. (2015). Fear and time: Fear speeds up the internal clock. *Behavioural*

Processes, 120, 135-140, ISSN 0376-6357, https://doi.org/10.1016/j.beproc.2015.09.014.

Foss, M., Liu, Y.-C, & Yarahmadian, S. (2022). An Evaluation of Mathematical Models and Stability Analysis of Learning Based on Reaction Kinetics. *International Journal on Engineering, Science, and Technology, 4*(4), 322-343.

Foss, M., & Liu, Y.-C. (2021). Creating solutions through project-based and experimental learning: A case study of the Concept Center. *International Journal of Engineering Education, 37*(6), 1630-1642.

Foss, M., & Liu, Y.-C. (2021) Developing creativity through project-based learning. *Proceedings of Wasatch Aerospace and Systems Engineering Mini-Conference*, virtual conference, April 15-16, 2021.

Foss, M., & Liu, Y.-C. (2022). Lessons Learned from 5 Years of Parent Daughter Engineering Outreach: Using Project-Based Learning to Introduce Families to Engineering Disciplines, *Proceedings of 2022 ASEE Annual Conference and Exposition*, Minneapolis, MN, June 26 – 29, 2022.

Foss, M., & Liu, Y.-C. (2020). Project-based learning center to bridge to students with technology. *Proceedings of 2020 Intermountain Engineering Technology and Computing (IETC),* Orem, UT, USA, September 2020.

Foss, M., Liu, Y.-C, & Yarahmadian, S. (2022). Project-based learning in engineering education in a virtual setting: A case study on materials and manufacturing process and applied statistics. *International Journal of Engineering Education, 39*(5A), 1377-1388.

Foss, M., & Liu, Y.-C. (2022). Promoting Sustainable Development Goals through Project-Based Learning: A Case Study of the Concept Center. *Proceedings of the 1st International Academy Conference on the Sustainable Development Goals*, October 5 – 7, 2022, Utah Valley University, Orem, UT, USA.

Gamez, D. (2014). The Measurement of Consciousness: A framework for the Scientific Study of Consciousness. *Frontiers in Psychology, 5*.

Illeris, K. (2018). *Contemporary Theories of Learning: learning theorists… In Their Own Words*. Routledge, London, UK.

Joye, S. (2017). Tuning the Mind in the Frequency Domain: Karl Pribram's Holonomic Brain Theory and David Bohm's Implicate Order. *Cosmos and History: The Journal of Natural and Social Philosophy, 13*(2), 166–184. Retrieved from https://cosmosandhistory.org/index.php/journal/article/view/601

Kleiner, J. (2020). Mathematical Models of Consciousness. *Entropy, 22*, 609. https://doi.org/10.3390/e22060609

Lake, J. I., & Meck, W. (2013). Differential effects of amphetamine and haloperidol on temporal reproduction: Dopaminergic regulation of attention and clock speed. *Neuropsychologia, 51*(2), 284-292. https://doi.org/10.1016/j.neuropsychologia.2012.09.014.

Logan, J.D. (2003). *Applied Mathematics.* John Wiley and Sons, Hoboken, NJ.

Lucas, C., Briders, S., Griffiths, T., & Gopnik, A. (2014). When children are better (or at least more open-minded) learners than adults: Developmental difference in learning the forms of causal relationships. *Cognition, 131*, 284-299.

Malapani, C., & Fairhurst, S. (2002). Scalar Timing in Animals and Humans. *Learning and Motivation, 33*(1), 156-176. https://doi.org/10.1006/lmot.2001.1105.

Namazi, H., & Kulish, K. (2012). A mathematical based definition of human consciousness. *Mathematics in Engineering, Science and Aeorospace, 3*, 189-198.

Ordahl, L. (1911). Consciousness in Relation to Learning. *The American Journal of Psychology, 22*(2), 158-213.

Pribram, K. H., & Carlton, E. H. (1986). Holonomic brain theory in imaging and object perception. Acta *Psychologica, 63*(2), 175-210,

Rudrauf, D., Bennequin, D., Granic, I., Landini, G., Friston, K., & Williford, K. (2017). A mathematical model of embodied consciousness. *Journal of theoretical biology, 428,* 106–131. https://doi.org/10.1016/j.jtbi.2017.05.032

Sarigiannidis, I., Grillon, C., Ernst, M., Roiser, J. P., & Robinson, O. J. (2020). Anxiety makes time pass quicker while fear has no effect. *Cognition, 197,* 104116. https://doi.org/10.1016/j.cognition.2019.104116.

Scheider, S., & Velmans, M. (2017). *The Blackwell Companion to Consciousness*, Second Edition. John Wiley and Sons, Hoboken, NJ.

Schneider, J. (2013). Remembrance of Things Past: A History of the Socratic Method in the United States. *Curriculum Inquiry, 43*(5), 613–40. http://www.jstor.org/stable/23524643.

Seth, A. K., Dienes, Z., Cleeremans, A., Overgaard, M., & Pessoa, L. (2008). Measuring consciousness: relating behavioural and neurophysiological approaches. *Trends in Cognitive Sciences, 12*(8), 314-321.

Tyler, C. (2020). Ten Testable Properties of Consciousness. *Frontiers in Psychology, 11*, 1144.

Wearden, J. H. (2013). The cognitive neuroscience of time perception: How psychological studies might help to dissect the timing system. *Neuropsychologia, 51*(2), 187-190.

Wearden, J. H. (2015). Passage of time judgements. *Consciousness and Cognition, 38,* 165-171, ISSN 1053-8100, https://doi.org/10.1016/j.concog.2015.06.005.

Author Information

Mary Foss

https://orcid.org/0000-0002-8919-9815

Weber State University

1447 Edvalson St. Dept 1802 Ogden, UT 84408

USA

Contact e-mail: *maryfoss@weber.edu*

Yucheng Liu

https://orcid.org/0000-0002-3463-7996

South Dakota State University

Crothers Engineering Hall 221, Box 2219

Brookings, SD

USA

Shantia Yarahmadian

https://orcid.org/0000-0002-8088-9772

Mississippi State University

410 Allen Hall

175 President's Circle P.O. Box MA

Mail Stop: 9715

Mississippi State, MS

USA