

Geometry for a Symmetric 2-Path Atom Interferometer Model

Yakubu Adamu

Department of Physics, University of Aberdeen, UK, y.adamu.18@abdn.ac.uk

Charles H.-T. Wang

Department of Physics, University of Aberdeen, UK

Abstract: It is often argued that the sensitivity of atom interferometer depends on the geometry of the interfering atom paths, conventionally, atom interferometer are usually configured to be sensitive to all deformations including tensor and scalar field deformations, it is a promising and robust tool for obtaining a highly sensitive and accurate measurements of gravitational signals, as such they are potentially capable of testing a wide range of fundamental physics questions including gravitational decoherence. Therefore, motivated by the recent search to improve the sensitivity of the next generation atom interferometer, we derived a broad class of equations that obeys a specific geometric configuration for the interfering paths for a 2-level atom interferometer model, and in doing so, we further analysed various configurations for the geometric interpretation, in addition to interferometric influence phase shift and a possible decoherence factor which could lead to a systematic theoretical framework for future sensing of weak forces, due to, for example, gravitational waves and light dark matter.

Keywords: Geometry, Interferometer, Phase shift, Two-level atom, Symmetry

Introduction

The science of atom interferometry has developed very rapidly in past few decades, making them a unique choice for future quantum sensing of weak forces, due to, for example gravitational wave and light dark matter. They are robust and potentially capable of detecting gravitational waves and scalar fields as well as ensuring little interaction with unwanted environmental influences, such as electric and magnetic fields [1]. However, for the next generation atom interferometer to surpass the previous sensitivity, it would require a significant improvement in order to test for many fundamental physics questions including gravitational decoherence. To this end, improving the sensitivity of atom interferometer and setting higher limits for the search for ultra- light dark matter can offer an unprecedented potential for new discoveries.

Therefore, in this paper, we derived a broad class of geometric equations for a symmetric 2-path atom interferometer model which obeys specific time configuration that would help in improving the sensitivity of the next generation atom interferometer. And in doing so, we consider the fact that the sensitivity of atom interferometer would depend on the geometric configuration of the two atom paths, since they operate by splitting the wave-function of an atom onto two spatially separated paths and subsequently measuring their phase difference[2], Here, we show a proof of this fact by constructing a phase functional for a possible decoherence factor for the 2-path model in light of the geometric interpretation of the model.

First, we consider equation (5.20) of ref. [3], which represent the interference effect of the 2-path atom interferometer with translational degrees of freedom.

$$\begin{aligned} \check{d}(\mathbf{k}, t) = & \hbar\Omega_{\rho} e^{-i\omega t} \int_0^t dt' e^{-i(kr_1(t') - \omega t')} \\ & - \hbar(A\Omega_{\rho} - a\omega_{\rho}) e^{-i\omega t} \int_0^t dt' e^{-i(kr_2(t') - \omega t')} \end{aligned} \quad (1.1)$$

Equation (1.1) represent the interference effect of the 2-path of atom interferometer to be described, where $r_1(t')$ and $r_2(t')$ are the classical path of the atom, $\hbar\omega_{\rho}$ is a kinetic term is described the unperturbed transition frequency from ground state to the excited state and $\hbar\Omega_{\rho}$ is the total ionisation energy (mass-energy transition) of the atom and a and A are some coupling coefficient terms

The 2-Path Atom Interferometer Model

For the model presented here, we consider a quantum particle in 2-level atom model, which experience a linearly varying potential with a translational degree of freedom, in the case of prescribed atom path $r(t)$, and described by the Hamiltonian density

$$\mathcal{H}_{SI} = \mathcal{H}_0 + \delta(x - r(t)) \hbar[\omega_0(1 + a\phi)|e\rangle\langle e| - A\Omega_0\phi] \quad (1.2)$$

This follows the parameters described by Equation.(1.1) where \mathcal{H}_{SI} is system interaction Hamiltonian density, \mathcal{H}_0 is a kinetic term and some coupling coefficients a and A for the total mass-energy transition of the atom.

For the sake of simplicity, we assumed that a quantum particle which is initially ($t' = 0$) will split into the spatially separated classical two paths $r_1(t')$ and $r_2(t')$ after been kicked by a photon momentum and recombines at ($t' = t$). The initial quantum states associated with the two paths are given respectively by $|\Psi_1(0)\rangle$ and $|\Psi_2(0)\rangle$ so that the total initial state takes the form

$$\Psi(0) = \frac{1}{\sqrt{2}}(|\Psi_1(0)\rangle + |\Psi_2(0)\rangle) \quad (1.3)$$

corresponding to the density matrix

$$\rho(0) = \frac{1}{\sqrt{2}}(|\Psi_1(0)\rangle\langle\Psi_1(0)| + |\Psi_2(0)\rangle\langle\Psi_2(0)| + |\Psi_1(0)\rangle\langle\Psi_2(0)| + |\Psi_2(0)\rangle\langle\Psi_1(0)|) \quad (1.4)$$

By using (1.2), we can then define the states with respect to the atom paths as follows:

$$|\Psi_1(0)\rangle = |g\rangle |r_1(t')\rangle \quad (1.5)$$

$$|\Psi_2(0)\rangle = |g\rangle |r_2(t')\rangle \quad (1.6)$$

Then from (1.5) and (1.6) we have:

$$q_1(x, t) = \delta(x - r_1(t')) \hbar A \Omega_0 \phi \quad (1.7)$$

$$q_2(x, t) = \delta(x - r_2(t')) \hbar (A \Omega_0 \phi - a \omega_0 \phi) \quad (1.8)$$

And hence,

$$d(x, t) = \delta(x - r_1(t')) \hbar A \Omega_0 \phi - \delta(x - r_2(t')) \hbar (A \Omega_0 \phi - a \omega_0 \phi) \quad (1.9)$$

The perspective for this model is that when laser light interact with atomic particle in the interferometer, such interaction (light-atom) will coherently drives the atomic populations in the ground to an excited states in an oscillatory manner, putting the atoms into an equal-probability superposition of both ground and excited states.

Taking into account the photon momentum, caused by laser pulses which create a superposition of momentum states, the excited part of the superposition will then move away from the ground state, and thus travel simultaneously along different paths as described in Fig 1.

Although paths may differ in height, but for this model we assumed that they are symmetric such that the 2 part in Eq (1.1) will meet continuously, in a way that the resulting interference phase will depend on the gravitational potential difference between the trajectories [4], and by so doing the interferometer becomes a very sensitive gravity sensor.

Geometry for Interferometer Model

To describe the interferometer geometry for the model, let us consider Fig. 1, which shows a 2-path r_1 and r_2 configuration (colour-lines), this form an interferometer to be described by equation (1.1). Here, we assumed that the two arms, r_1 and r_2 , of our interferometer will experience accelerations $r_1(t')$ and $r_2(t')$ respectively at time t , which we shall define from $t' = 0$ at the initial state of the interferometer as shown in Fig. 1.

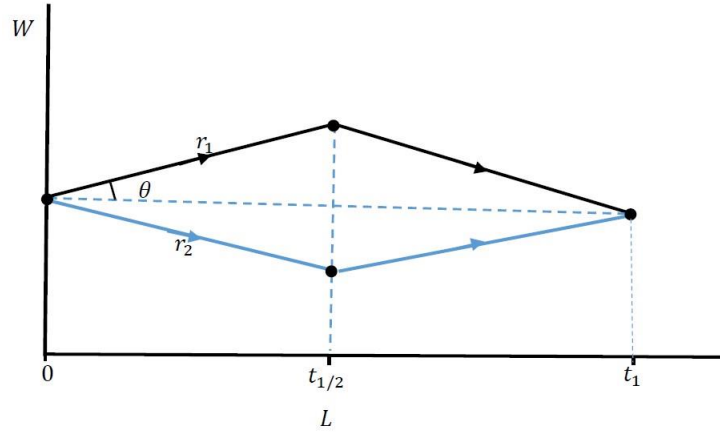


Figure 1. Diagram of a Symmetric 2-Path Atom Interferometry in The Absence of Gravity. the 2-Paths r_1 and r_2 Satisfies the Symmetry Properties Described in Eq. 1.1. Such That The Two-Paths Recombines at (t') Continuously.

The starting point of this approach is based on the classical action of atom-light interaction. We assume that the atom is in a inertial frame of reference at $t' = 0$ (See Fig 2), and the action of laser light-atomic interactions would affect the motion of the atom, which would leads to linear displacements of the atom into different trajectories of the symmetric 2-paths. The classical action allows us to calculate the interferometer geometry and the exact interferometer phase shift for the moder under consideration. .

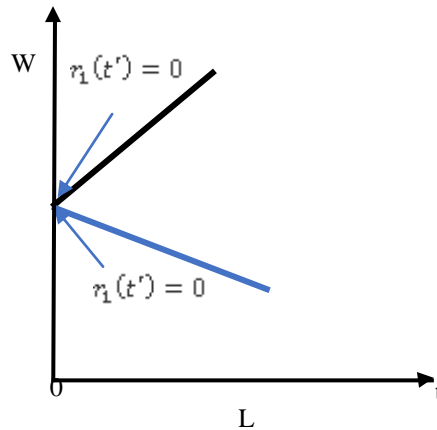


Figure 2. The Acceleration Kick on The Quantum Particle along The Paths Due to Photon Momentum. We Consider That the Particle is Initially at Rest Would Experience a Linear Acceleration Having Been Kicked by Action of Light-Atom Interaction.

Therefore, to calculate the interferometer geometry, the following expressions describe the symmetric 2-path configuration for the classical paths r_1 and r_2 with length L and width W . In this expression we consider only the integral of the potential term, such that the two paths are governed by coordinate components of the specified paths.

Hence,

$$r_1(t) = \begin{cases} (\cos\theta vt, \sin\theta vt, 0), & \text{if } 0 < t < t_{1/2} \\ \left(\frac{L}{2} + \cos\theta vt, \frac{W}{2} - \sin\theta vt, 0\right), & \text{if } t_{1/2} < t < t_1 \end{cases} \quad (1.10)$$

$$r_2(t) = \begin{cases} (\cos\theta vt, -\sin\theta vt, 0), & \text{if } 0 < t < t_{1/2} \\ \left(\frac{L}{2} + \cos\theta vt, -\frac{W}{2} + \sin\theta vt, 0\right), & \text{if } t_{1/2} < t < t_1 \end{cases} \quad (1.11)$$

where $\cos\theta vt, \sin\theta vt$ etc are the coordinate component along $r_1(t)$ and $r_2(t)$ paths respectively and

$$\tan \theta = W/L \text{ and } vt = \sqrt{W^2 + L^2}$$

Now, evaluating equation (1.1) using the explicit components of $r_1(t)$ and $r_2(t)$ for the total time sequence labelled by (t) , we have the following

$$\begin{aligned} \hat{d}(\mathbf{k}, t_1) &= \hbar \Omega_0 e^{-i\omega t_1} \int_0^{t_1/2} dt' e^{-i(k_x r_1(t') - \omega t')} \\ &\quad - \hbar(A\Omega_0 - \alpha\omega_0) e^{-i\omega t_1} \int_0^{t_1/2} dt' e^{-i(k_x r_2(t') - \omega t')} \\ &\quad + \hbar \Omega_0 e^{-i\omega t_1} \int_{t_1/2}^{t_1} dt' e^{-i(k_x r_1(t) - \omega t)} \\ &\quad - \hbar(A\Omega_0 - \alpha\omega_0) e^{-i\omega t_1} \int_{t_1/2}^{t_1} dt' e^{-i(k_x r_2(t') - \omega t')} \\ &= \hbar \Omega_0 e^{-i\omega t_1} \int_0^{t_1/2} dt' e^{-i\Omega_+ t} \\ &\quad - \hbar(A\Omega_1 - \alpha\omega_0) e^{-i\omega t_1} \int_0^{t_1/2} dt' e^{-i\Omega_- t} \\ &\quad + \hbar \Omega_0 e^{-i(k_x L/2 + k_x W/2)} e^{-i\omega t_1} \int_{t_1/2}^{t_1} dt' e^{-i\Omega_+ t} \\ &\quad - \hbar \Omega_1 e^{-i(k_x L/2 - k_x W/2)} e^{-i\omega t_1} \int_{t_1/2}^{t_1} dt' e^{-i\Omega_- t} \end{aligned} \quad (1.12)$$

Where

$$\begin{aligned} \Omega_+ &= k_x v \cos \theta + k_y v \sin \theta - \omega \\ \Omega_- &= k_x v \cos \theta - k_y v \sin \theta - \omega \\ \Omega_1 &= \Omega_0 - \frac{\alpha}{A} \omega_0 \end{aligned} \quad (1.13)$$

And the effective wave vector $\mathbf{k} = k_x = k_y$. Then, substituting the values of Ω_+ and Ω_- respectively into (1.12), and using exponential rule, it follows that

$$\begin{aligned} \hat{d}(\mathbf{k}, t) &= i\hbar A \Omega_0 e^{-i\omega t_1} \frac{(e^{-i\Omega_+ t_1/2} - 1)}{\Omega_+} \\ &\quad - i\hbar A \Omega_1 e^{-i\omega t_1} \frac{(e^{-i\Omega_- t_1/2} - 1)}{\Omega_-} \\ &\quad + i\hbar A \Omega_0 e^{-i(k_x L/2 + k_x W/2)} e^{-i\omega t_1} \frac{(e^{-i\Omega_+ t_1} - e^{-i\Omega_+ t_1/2})}{\Omega_+} \\ &\quad - i\hbar A \Omega_1 e^{-i(k_x L/2 - k_x W/2)} e^{-i\omega t_1} \frac{(e^{-i\Omega_- t_1} - e^{-i\Omega_- t_1/2})}{\Omega_-} \end{aligned} \quad (1.14)$$

The value of $\hat{d}(\mathbf{k}, t)$ using (1.14) represents the interference effect due the geometric configuration for the 2-path interferometer. The diamond geometry described in figure 1. can be used to maximize the sensitivity of the

interferometer phase to potential gradients. Therefore, by varying the size and length of the diamond shape, the interferometer will reveal the phase evulsion and coherence of the states with respect to the number of shifts. The changes in the phase evolution can be interpreted as the gradient strength along the 2- paths. This configuration (diamond geometry) is suitable to measure time-controllable gradients in a differential scheme [5].

Phase functional for 2-path Model

The phase functional for the atom interferometer model presented here is calculated based on the classical action of atom-light interaction. We assumed that the system is coupled to a stationary reservoir and in a Gaussian state, so that the dissipation kernel

$$\mathcal{D}(x-x') = \frac{1}{\hbar} \{ \langle \phi(x), \phi(x') \rangle \}_R \quad (1.15)$$

And the noise kernel

$$\mathcal{N}(x-x') = \frac{1}{\hbar} \{ \langle \phi(x), \phi(x') \rangle \}_R \quad (1.16)$$

can be introduced by using Eq.1.4 being the reduced density matrix of the matter at the initial time ($x' = 0$). Thus, by introducing both the dissipation and noise kernel

$$\begin{aligned} \rho(x) = T_-^Q \left(\exp \left[\int_0^4 d^4 x^4 \mathcal{L}_s^{s8lf}(x') \right. \right. \\ \left. \left. + \frac{1}{2\hbar} \int_0^4 d^4 x' \int_0^4 d^4 x'' \{ i\mathcal{D}(x'-x'') (Q_+(x') - Q_-(x')) (Q_+(x') - Q_-(x')) \right. \right. \\ \left. \left. + \mathcal{N}(x'-x'') (Q_+(x') - Q_-(x')) (Q_+(x') - Q_-(x')) \} \right] \right)_{\rho(0)} \end{aligned} \quad (1.17)$$

This leads to the influence phase functional

$$\begin{aligned} i\Phi[Q_+, Q_-] = \int_0^4 d^4 x^4 \mathcal{L}_s^{s8lf}(x') \\ + \frac{1}{\hbar^2} \int_0^4 d^4 x' \int_0^4 d^4 x'' \{ i\mathcal{D}(x'-x'') (Q_+(x') - Q_-(x')) (Q_+(x') - Q_-(x')) \\ + \mathcal{N}(x'-x'') (Q_+(x') - Q_-(x')) (Q_+(x') - Q_-(x')) \} \end{aligned} \quad (1.18)$$

with a density matrix given by

$$\rho(x) = T_-^Q \exp(i\Phi[Q_+, Q_-])_{\rho(0)}$$

Now, If we assume that

$$\begin{aligned} Q(x) &= |\Psi_1(0)\rangle \approx q_1(x) |\Psi_1(0)\rangle \\ Q(x) &= |\Psi_2(0)\rangle \approx q_2(x) |\Psi_2(0)\rangle \end{aligned} \quad (1.19)$$

And neglecting \mathcal{L}_s^{s8lf} , it follows from Eq. 1.4 that

$$\rho(x) \approx \frac{1}{2} (|\Psi_1(0)\rangle \langle \Psi_1(0)| + |\Psi_2(0)\rangle \langle \Psi_2(0)| + \vartheta^\theta |\Psi_1(0)\rangle \langle \Psi_2(0)| + \vartheta^{-\theta} |\Psi_2(0)\rangle \langle \Psi_1(0)|) \quad (1.20)$$

were

$$\begin{aligned} i\Phi[q_1, q_2] = -\frac{1}{2\hbar} \int_0^t d^4 x' \int_0^{t'} d^4 x'' \{ i\mathcal{D}(x'-x'') (q_1(x') - q_2(x')) (q_1(x'') + q_2(x'')) \\ + \mathcal{N}(x'-x'') (q_1(x') - q_2(x')) (q_1(x'') - q_2(x'')) \} \end{aligned} \quad (1.21)$$

is the effective influence phase functional for the 2-path model and the associated decoherence functional is given by the real part of $i\Phi[q_1, q_2]$.

Conclusion

In summary, we have derived some broad class of equations that represents a geometric factor for a symmetric

2-path atom interferometer model. we have also proved that the model presented here has the capability to measure time-controllable gradients thereby achieving a solution that can improve the sensitivity of atom interferometer. Furthermore, the symmetric two-path (diamond shape) which is expressed as a geometric equation is suitable for a 2-path atom interferometer model using the two-level atom system. The equation established here would allow us to better engineered an effective interferometer scheme which would laid both the theoretical and experimental foundation that are needed in understanding the concepts and theories of applications for sensing weak forces for example, gravitational wave.

Acknowledgements

The Authors wish to thank Afifurrahman and D. Rodriguez for very useful discussions. We acknowledge support from colleagues at the Department of Physics University of Aberdeen UK.

References

- [1]. Bongs, K., Holynski, M., Vovrosh, J., Bouyer, P., Condon, G., Rasel, E., Schubert, C., Schleich, W.P., and Roura, A. (2019) *Taking atom interferometric quantum sensors from the laboratory to real-world applications*. *Nat Rev Phys* **1**, 731–739. <https://doi.org/10.1038/s42254-019-0117-4>
- [2]. Cronin, A.D., Schmiedmayer, J., and Pritchard, D.E. (2009) *Optics and interferometry with atoms and molecules*. *Rev Mod Phys* **81**:1051–1129.
- [3]. Wang, C. H.-T. and Adamu, Y. (2019) Decoherence in an atom interferometer due to light dark matter. ResearchGate, Reprint of Unpublish Manuscript.
- [4]. Neder, I., Heiblum, M., Levinson, Y., Mahalu, D. and Umansky, V. (2006) *Unexpected Behavior in a Two-Path Electron Interferometer*, *Phys. Rev. Lett.*, vol. 96, no. 1, p. 016804.
- [5] Steffen, A., Alberti, A., Alt, W., Belmechri, N., Hild, S., Karski, M., Widera, A., and Meschede, D. (2012) *Digital interferometer with single atoms*, *Proceedings of the National Academy of Sciences* Jun 2012, **109** (25) 9770-9774; DOI: 10.1073/pnas.1204285109