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Approximation Procedure for Determining Oscillation Period of Frame Structure

Faris Trešnjo
Interprojekt d.o.o. Mostar, Bosnia and Herzegovina, faris.tresnjo@gmail.com

Majudin Muratović
Dzemal Bijedic University of Mostar, Bosnia and Herzegovina

Abstract: Dynamic analysis can be used to find dynamic displacements, time history, and the frequency content of the load. One analysis technique for calculating the linear response of structures to dynamic loading is a modal analysis. In modal analysis, we decompose the response of the structure into several vibration modes. A mode is defined by its frequency and shape. Structural engineers call the mode with the shortest frequency (the longest period) the fundamental mode. Holzer and Stodola's approximate methods for determining the forms and periods of oscillation for frame structures are presented in the paper. An approximation method, based on approximate relative stiffnesses of the storeys and the ground floor, is analyzed and proposed. The results obtained by the proposed approximate procedure do not greatly deviate from those obtained by more accurate calculations. It is therefore emphasized that the method can be used both in practice and for checking computer-based analysis of complex systems. At the end of the paper was given a comparison of the results obtained by approximate methods and some engineering software.

Keywords: Oscillation period, Frame structure, Approximate methods

Introduction

In the framework of seismic risk assessment and mitigation the estimation of fundamental period of buildings is an important issue both for design of new buildings and performance assessment of existing ones. Depending on mass and stiffness, the fundamental period is a global characteristic describing the behaviour of building under seismic loads. For this reason, it is easily and directly usable to determinate the global demands on a structure due to a given seismic input. Moreover, the estimation of fundamental period of buildings is useful to identify possible resonance phenomena between buildings and soil vibration [1].

Buildings oscillate during earthquake shaking. The oscillation causes inertia force to be induced in the building. The intensity and duration of oscillation, and the amount of inertia force induced in a building depend on features of buildings, called their dynamic characteristics, in addition to the characteristics of the earthquake shaking itself. The important dynamic characteristics of buildings are modes of oscillation and damping. A mode of oscillation of a building is defined by associated Natural Period and Deformed Shape in which it oscillates [2].

Natural Period T of a building is the time taken by it to undergo one complete cycle of oscillation. It is an inherent property of a building controlled by its mass m and stiffness k. The reciprocal (1/T) of natural period of a building is called the Natural Frequency fn; its unit is Hertz (Hz).

Buildings with larger mass m and with smaller stiffness k have larger natural period than light and stiff buildings. Usually, natural periods (T) of 1 to 20 storey normal reinforced concrete and steel buildings are in the range of 0.05 - 2.00s. Resonance will occur in a building, only if frequency at which ground shakes is steady at or near any of the natural frequencies of building and applied over an extended period of time.

Approximation Procedure for Determining Oscillation Period of Frame Structure

In the paper are presented Holzer and Stodola's approximate methods for determining the forms and periods of oscillation for frame structures. An approximation method, based on approximate relative stiffnesses of the storeys and the ground floor, is analyzed and proposed. The results obtained by the proposed approximate procedure do not greatly deviate from those obtained by more accurate calculations. It is therefore emphasized that the method can be used both in practice and for checking computer-based analysis of complex systems.

Stodola's Method

A method of calculating the deflection of a uniform or non-uniform beam in free transverse vibration at a specified frequency, as a function of distance along the beam, in which one calculates a sequence of deflection curves each of which is the deflection resulting from the loading corresponding to the previous deflection, and these deflections converge to the solution [3].

The differential equation of the system for free undamped oscillations is:

$$[M]\cdot\{\ddot{x}\}+[K]\cdot\{x\}=\{0\}$$

We get the solution of this equation if we assume a harmonic function:

$\{x\}=\{v\}\sin\ \omega t$

where is:

$\{v\}$ – indeterminate displacement vector

If we derive the equation twice, put it in the differential equation of the system and after arranging the equation we will get the following:

$[K]\{v\}=\omega^2[M]\{v\}$

The product of the flexibility matrix and the mass matrix gives us a dynamic matrix D:

$[D]=[K]^{-1}\cdot[M]$

And now, differential equation has form:

$[D]\{v\}=\dfrac{1}{\omega^2}\{v\}$

If we make a substitution:

$\dfrac{1}{\omega^2}=\lambda$

then we get a general expression suitable for iteration:

$\boxed{[D]\{v\}=\lambda\{v\}}$

For example, the first step of the iteration would be:

$[D]\{v^0\}=\{\overline{v}^1\}$

Where the vector $\{v^0\}$ is a assumed vector and the vector $\{\overline{v}^1\}$ is the current displacement vector obtained by multiplying the dynamic matrix and the assumed vector.

The second step of the iteration would be:

$[D]\{v^1\}=\{\overline{v}^2\}$

Where the vector $\{v^1\}$ is the obtained displacement vector from the first iteration step, and the vector $\{\overline{v}^2\}$ is the new current displacement vector obtained by multiplying the dynamic matrix and the displacement vector from the first iteration step.

We repeat the iteration procedure until the previous and next vectors completely coincide. With this iteration, we get the first form of oscillation of the construction.

Holzer's Method

Holzer's method of finding natural frequency of a multi-degree of freedom system. Holzer's Method. This method is an iterative method and can be used to determine any number of frequencies for a multi degrees of freedom (DOF) of system.

An important advantage of the Holzer method is that the natural vibration frequencies can be determined independently of each other. The procedure of solving problems using the Holzer method is to use a system with reduced concentrated masses. Frame constructions with rigid floor beams have one dynamic degree of freedom of movement per mass. We put the displacement amplitude at the top of the object to $u = 1,0$. According to this standardized amplitude, we determine the amplitudes of the displacement of the next mass (floor) for the selected circular vibration frequency. We study the obtained results for displacement at the site of the foundation of the object. If $u_t \neq 0$, then there is a displacement at the site of the foundation of the object, and the procedure needs to be repeated with a new assumed frequency. We have to repeat the procedure until we get $u_t = 0$.

The frequency corresponding to such a form of vibration is the corresponding frequency ω, and according to it we determine the period of vibration of the studied structure:

$$T = \frac{2\pi}{\omega}.$$

However, the determination of one oscillation frequency is independent of the other, each obtained according to the described procedure [4].

Theoretical Investigations of Holzer's and Stodola's Method for Multi Storey Framed Structures

We need to determine the first natural form of oscillation of the structure, the natural frequency and the periods of oscillation of the structure, taking the beams as absolutely rigid, where the columns have their own stiffness, but negligible mass.

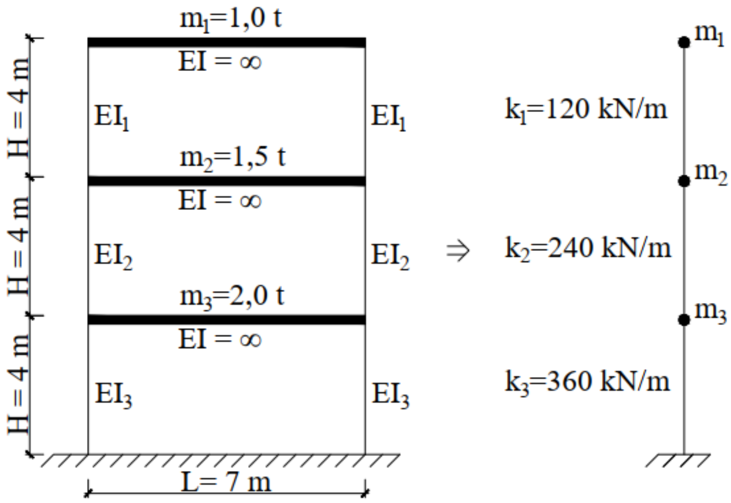


Figure 1. Framed Structure

Stodola's Method

Mass matrix:	Stiffness matrix:	Dynamic matrix:
$[M]=\begin{bmatrix}1 & 0 & 0\\ 0 & 1,5 & 0\\ 0 & 0 & 2\end{bmatrix}$	$[K]=\begin{bmatrix}120 & -120 & 0\\ -120 & 360 & -240\\ 0 & -240 & 600\end{bmatrix}$	$[D]=[K]^{-1}[M]=\begin{bmatrix}15,28 & 10,41 & 5,56\\ 6,94 & 10,41 & 5,56\\ 2,78 & 4,16 & 5,56\end{bmatrix}$

Table 1. Iterative Procedure for the First Form of Oscillation

	v^0	v^1	v^2	v^3
the first mode of vibration	$\begin{Bmatrix}1\\1\\1\end{Bmatrix}$	$\begin{Bmatrix}1,000\\0,73\\0,40\end{Bmatrix}$	$\begin{Bmatrix}1,00\\0,67\\0,32\end{Bmatrix}$	$\begin{Bmatrix}1,00\\0,65\\0,31\end{Bmatrix}$

$[D]\{v\} =$	$\begin{Bmatrix} 31,25 \\ 22,91 \\ 12,50 \end{Bmatrix} \cdot 10^{-3}$	$\begin{Bmatrix} 25,10 \\ 16,76 \\ 8,04 \end{Bmatrix} \cdot 10^{-3}$	$\begin{Bmatrix} 24,04 \\ 15,69 \\ 7,35 \end{Bmatrix} \cdot 10^{-3}$	$\begin{Bmatrix} 24,04 \\ 15,69 \\ 7,35 \end{Bmatrix} \cdot 10^{-3}$
$\lambda =$	$31,25 \cdot 10^{-3}$	$25,10 \cdot 10^{-3}$	$24,04 \cdot 10^{-3}$	$24,04 \cdot 10^{-3}$

	v^4	v^5	v^6	
the first mode of vibration	$\begin{Bmatrix} 1,000 \\ 0,649 \\ 0,303 \end{Bmatrix}$	$\begin{Bmatrix} 1,000 \\ 0,648 \\ 0,301 \end{Bmatrix}$	$\begin{Bmatrix} 1,000 \\ 0,648 \\ 0,301 \end{Bmatrix}$	
$[D]\{v\} =$	$\begin{Bmatrix} 23,72 \\ 15,38 \\ 7,16 \end{Bmatrix} \cdot 10^{-3}$	$\begin{Bmatrix} 23,70 \\ 15,36 \\ 7,15 \end{Bmatrix} \cdot 10^{3}$		
$\lambda =$	$23,72 \cdot 10^{-3}$	$23,70 \cdot 10^{-3}$		

We repeat the iteration procedure until the previous and next vectors completely coincide. With this iteration, we get the first form of oscillation of the construction.

And from Table 1, the first form of oscillation is $\begin{Bmatrix} \mathbf{1,000} \\ \mathbf{0,648} \\ \mathbf{0,301} \end{Bmatrix}$.

After determining the first form of oscillation, we can calculate the first natural circular frequency and oscıllatıon perıod:

$$\frac{1}{\omega^2} = \lambda\{v\}$$

$$\omega_1 = \frac{1}{\sqrt{\lambda}} = \frac{1}{\sqrt{23,70 \cdot 10^{-3}}}$$

$$\omega_1 = 6,50 \frac{\text{rad}}{\text{s}}$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{6,50} = 0,967 \text{ s}$$

Holzer’s Method

The first step of Holzer's method is to assume the first frequency.

<u>I assumption:</u>

The displacement of the mass m_1, that is, the displacement at the top of the frame by this method is $u_1 = 1,0$.

The inertia force influence on the mass m_1 is obtained according to the expression:

$$H_1 = m_1 \cdot \omega_a^2 \cdot u_1 = 1 \cdot 5^2 \cdot 1 = 25 \text{ kN}$$

From the static equilibrium conditions, the internal transverse force T_1 is determined as a reaction to the obtained inertia force.

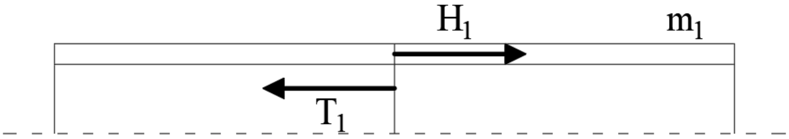


Figure 2. Transverse Force and Inertia Force

$T_1 = H_1 = 25 \text{ kN}$

After determining the transverse force, the deformation of the highest floor of the structure is determined, according to the expression:

$$\Delta u_1 = \frac{T_1}{k_1} = \frac{25}{120} = 0,208 \text{ m}$$

Therefore, the displacement of the second floor is:

$$u_2 = u_1 - \Delta u_1 = 1,00 - 0,208 = 0,792 \text{ m}$$

The inertia force of the second floor is:

$$H_2 = m_2 \cdot \omega_a^2 \cdot u_2 = 1,5 \cdot 5^2 \cdot 0,792 = 29,70 \text{ kN}$$

Again, the transverse force below the second floor is determined from the conditions of static equilibrium:

$$T_2 = H_1 + H_2 = 25,00 + 29,70 = 54,70 \text{ kN}$$

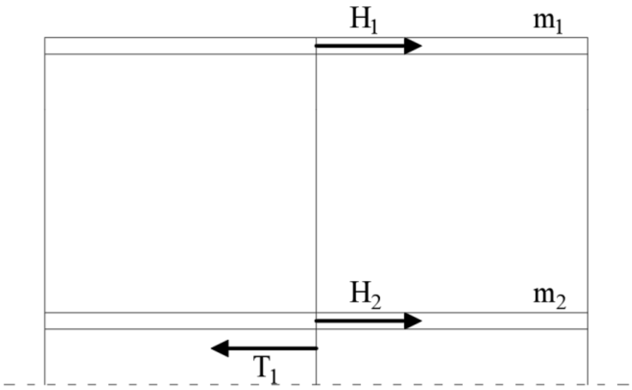


Figure 3. Transverse Force and Intertia Force below the Second Floor

The displacement of the second floor of the structure is:

$$\Delta u_2 = \frac{T_2}{k_2} = \frac{54,70}{240} = 0,228 \text{ m}$$

Analogous to the previous floor, the procedure continues for the first floor of the structure.
The displacement of the first floor of the structure is:

$$u_3 = u_2 - \Delta u_2 = 0,792 - 0,228 = 0,564 \text{ m}$$

The inertia force of the first floor is:

$$H_3 = m_3 \cdot \omega_a^2 \cdot u_3 = 2,0 \cdot 5^2 \cdot 0,564 = 28,20 \text{ kN}$$

The transverse force below the firs floor is:

$$T_3 = H_1 + H_2 + H_3 = 25,00 + 29,70 + 28,20 = 82,90 \text{ kN}$$

The displacement of the first floor of the structure is:

$$\Delta u_3 = \frac{T_3}{k_3} = \frac{82,90}{360} = 0,230 \text{ m}$$

The displacement of the foundation is obtained according to the expression:

$$u_t = u_3 - \Delta u_3 = 0,564 - 0,230 = 0,334 \neq 0 \text{ m}$$

We didn't get $u_t = 0$. Because the displacement of the foundation is positive ($u_t > 0$), we have to repeat the procedure with a higher frequency.

In a case, we got a negative displacement of the foundation ($u_t < 0$), then the procedure would be repeated with a smaller frequency.

<u>II assumption:</u>

$$\boxed{\omega_b = 6,4 \frac{\text{rad}}{\text{s}}}$$

Analogous to the first assumed frequency, the following values are obtained:

Table 2. Results for Second Assumption $\omega_b = 6,4\frac{\text{rad}}{\text{s}}$

H_1	$40,96$ kN
Δu_1	$0,341$ m
u_2	$0,659$ m
H_2	$40,49$ kN
T_2	$81,45$ kN
Δu_2	$0,339$ m
u_3	$0,320$ m
H_3	$26,21$ kN
T_3	$107,66$ kN
Δu_3	$0,299$ m
u_t	$0,021 \neq 0$ m

We are closer to the solution with the second assumed frequency, but we didn't get $u_t \approx 0$.

<u>III assumption:</u>

$$\boxed{\omega_c = 6,48\frac{\text{rad}}{\text{s}}}$$

Table 3. Results for third assumption $\omega_b = 6,4\frac{\text{rad}}{\text{s}}$

H_1	$41,99$ kN
Δu_1	$0,350$ m
u_2	$0,659$ m
H_2	$40,94$ kN
T_2	$82,93$ kN
Δu_2	$0,346$ m
u_3	$0,304$ m
H_3	$25,53$ kN
T_3	$108,46$ kN
Δu_3	$0,301$ m
u_t	$0,003 \approx 0$ m

The displacement of the foundation is approximately equal to zero, so the first oscillation frequency is:

$$\boxed{\boldsymbol{\omega_1 = \omega_c = 6,48\frac{\text{rad}}{\text{s}}}}$$

The first mode of vibration $\begin{Bmatrix} \mathbf{1,00} \\ \mathbf{0,650} \\ \mathbf{0,304} \end{Bmatrix}$.

Period of oscillation is:

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{6,48} = 0,970\,\text{s}$$

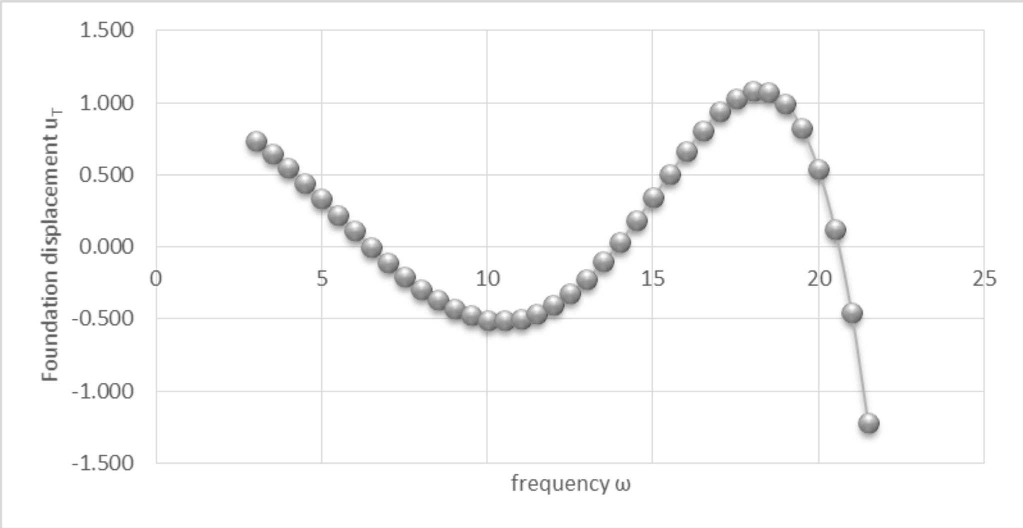


Figure 4. Dependence of Frequency and Displacement in the Foundation

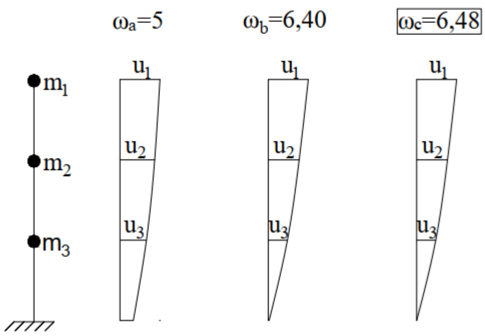


Figure 5. The First Mode of Vibration for I, II and III Assumption

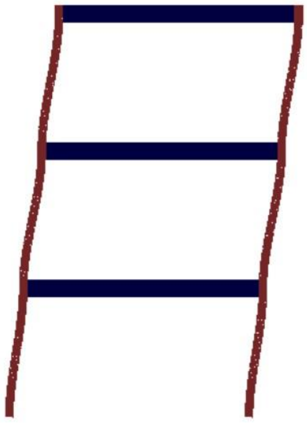
Comparing Results with Engineering

When we have structures where the deformations of the floor affect the movement of the system, then the approximate methods do not give accurate results and the calculation must be performed using a computer. In our case, the stiffness of the beam (floor) was taken to be infinite, i.e. it is much higher than the stiffness of the columns, so we could perform the calculation by approximate methods because the deformations of the beam do not affect the displacement of the system.

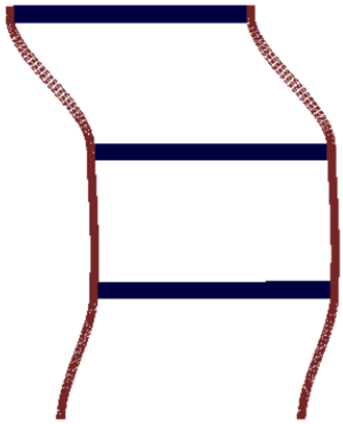
We used engineering software Tower 8 and the American software ELS to calculate the period of oscillation for the same characteristics of the constructions and we get:

Table 4. Results from engineering softwares Tower 8 and ELS

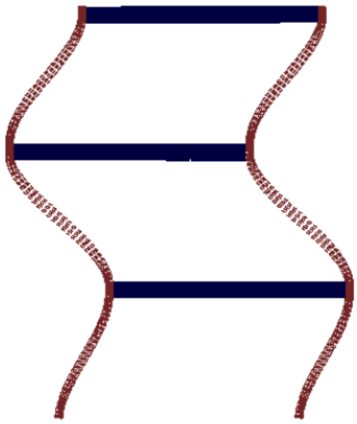
	Holzer's method	Stodola's method	Tower 8	ELS
Period of oscillation T_1	0,970	0,967	0,981	1,065
Period of oscillation T_2	0,452	0,452	0,454	0,472
Period of oscillation T_3	0,305	0,305	0,306	0,307



The First Mode of Vibration



The Second Mode of Vibration



The Third Mode of Vibration

Conclusion

Table 4. shows that the first oscillation period obtained in the Tower software deviates by about 1% compared to the result obtained using approximate methods, while the slightly larger deviation in the ELS software is about 9%.

The deviation of the values for the second and third oscillation periods obtained using the software and approximate methods is much smaller compared to the first oscillation period.

The application of approximate methods requires only a few basic data on the construction of the frame, so these approximate methods are suitable for determining their own periods and forms of oscillation in previous analyzes of structures or for controlling the calculation of complex systems using computers when is bigger possibility of mistake.

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