

Experimental Validation of Voltage Regulation in Buck Converters through Fractional-Order PID Approximation

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Abstract: Viability of a fractional-order PID approximation regulating voltage in buck converters through a single control loop is investigated. Fractional calculus approach is suggested due to it exhibits good robustness against parameter variations. The non-integer approach is integrated in the control strategy through a Laplacian operator biquadratic approximation to generate a flat phase curve in the system closed-loop frequency response, which results in the generation of the iso-damping characteristic. The synthesis and tuning process consider both robustness and closed-loop requirements to ensure a fast and stable regulation characteristic. Experimental data obtained with the resulting controller, which was easily implemented through RC circuits and OPAMPs in adder configuration, confirmed its effectiveness. Superiority of proposed approach, which is determined through a comparison with typical PID controllers, confirms its viability to be used in highly efficient converters, such as Silicon-Carbide ones.

Keywords: Fractional-order PID, DC-DC converters, Frequency response, Iso-damping property

Introduction

DC-DC Buck converter is one of the most studied from elementary converters due to its wide range of applications, which include control velocity of DC motors and regulated source of power (Soriano-Sánchez et al., 2020) or photovoltaic generation system (Sitbon et al., 2020), to mention a few. As a result of the attracted interest, a significant body of knowledge on control techniques to regulate voltage in Buck converters has been established. Among the most relevant can be listed fuzzy logic control (Alim et al., 2020), sliding mode control (Zhang et al., 2020), passivity-based control (Hernández-Guzmán et al., 2020), pole placement (Junior et al., 2020) and the well-known PI/PID control (Hekimoğlu & Ekinci, 2020; Nishat et al., 2020).

To improve modeling, performance/efficiency or reducing conduction losses, a control strategy with non-integer approach was suggested. Fractional calculus was considered due to its ability to describe better dynamical systems and its proven robustness against parameter variations, mainly. Many non-integer order control strategies have already been designed and used successfully in power converter, thus contributing to the body of knowledge of Control Theory and Power Electronics. Some of the most recent results are the following: in (Saleem et al., 2002) a self-adaptive control strategy was suggested. A PID controller is first optimally tuned to later augment its integral/derivative modes with non-integer operators. A zero-mean Gaussian function of error and its derivative were used to update fractional orders, thus producing an online control law. Rapid transients, minimum transient recovery time, and minimal fluctuations were the improvements. A fractional-order PI controller to stabilize Buck converter transfer function was suggested in (Valele et al., 2020). For the controller non-integer structure, the Oustaloup approximation was used, whose parameters were tuned through Nelder-Mead algorithm. Stability, robustness against parameter variations and acceptable performance indicators were the improvements. In (Jain et al., 2020) a fractional-order internal model controller was suggested. The control strategy is the combination of a PID controller and a non-integer order low pass filter. The internal mode control allowed access to the control law so it could be fed to the non-minimum phase part of the system and the low pass filter ensures robustness of the system. Transient performance and robustness against uncertainties/perturbations were improved. In (Abdelmalek et al., 2020) a sliding mode approach with two extra states augmentation was suggested. The control strategy considered a nonlinear observer for state estimations. Controller parameters are obtained through Particle Swarm Optimization algorithm. A balanced dynamical performance against disturbances and Lyapunov stability were guaranteed. A fractional-order sliding mode controller with back-stepping approach and parameters optimally tuned was suggested in (Delavari & Naderian, 2019). Robustness in presences of uncertainties, unmodeled dynamics and non-linear loads, steady-state error reduction and perturbation rejection were the improvements. In (Saleem et al., 2019) and (Farsizadeh et al.,

2019) fractional-order PI/PID controllers in combination with fuzzy-logic approach were suggested. The control strategy achieved voltage regulation via capacitor current to improve tracking performance, disturbance rejection and stability.

The described results confirmed that fractional-order calculus is an alternative to achieved different control objective in power converters. In spite of the resulting controllers were effective, the control strategies present the drawback of computational complexity, which was increased due to the integro-differential operator used or the need of using an optimization algorithm to compute controller parameters.

In this paper, the viability of a fraction-order PID controller approximation for voltage regulation in Buck converters is explored. The synthesis and tuning process is simple and considers both robustness and performance indicators. The frequency domain approach is integrated in the strategy and controller design through a biquadratic approximation that produces a flat phase response in the system closed-loop response. Experimental realization of the resulting controller, which is easy implemented through RC circuits and OPAMPs in adder configuration, confirmed its effectiveness. Moreover, controller viability to be used in highly efficient converters, such as Silicon-Carbide ones, was determined quantitatively by comparing performance parameters with those obtained from typical PID controllers.

The paper is organized as follows: in *Preliminaries* a brief review on Buck converter and the method to integrate fractional calculus in the control strategy is described. In *Results* the controller design is described. Simulations and experimental results are provided in this section as well. In *Discussion* an analysis of the described method and both numerical and experimental results are provided. Some conclusions are provided in *Conclusion* section.

Preliminaries

In this section preliminaries on Buck converter model, its transfer function, and the methodology to approximate the fractional-order PID controller are briefly described.

Buck converter model

The DC-DC Buck converter is the elementary configuration that produces an average output voltage lower than the converter power supply. A Buck converter is composed of a DC voltage source V_g , an inductance L , a capacitor C , a resistor R and two complementary switches implemented with a MOSFET Q and a diode D , see Figure 1.

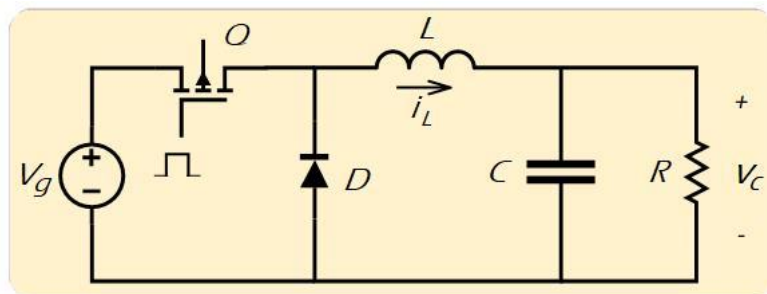


Figure 1. Buck Converter Electrical Diagram

By considering continuous conduction mode (CCM) and ideal components, the Buck converter average operation can be described by the continuous time-invariant model given as follows,

$$\begin{aligned} L \frac{di_L}{dt} &= \bar{D}V_g - v_c \\ C \frac{dv_c}{dt} &= i_L - \frac{v_c}{R} \end{aligned} \quad (1)$$

where $\bar{D} \in [0, 1]$ is the average of duty cycle D . The converter transfer function is given by,

$$G_p(s) = \frac{\frac{V_g}{CL}}{s^2 + \frac{1}{RC}s + \frac{1}{CL}} \quad (2)$$

whose parameters are given in Table 1.

Table 1. Parameter Values of Buck Converter in Figure 1

Element	Notation	Value
DC voltage	V_g	25 V
Capacitor	C	7 μ F
Inductor	L	2.7 mH
Resistance	R	10 Ω

In the remaining of this section, the method to approximate the fractional-order Laplacian operator will be described.

Laplacian Operator Biquadratic Approximation

The approximation of the integro-derivative operator $s^{\pm\alpha}$, where $0 < \alpha < 1$, can be achieved through the El-Khazali method, which consists in biquadratic modules with flat phase response (El-Khazali, 2013, El-Khazali, 2015). The operator s^α , i.e., with fractional derivative effect can be approximated as follows,

$$s^\alpha \approx T\left(\frac{s}{\omega_c}\right) = \frac{a_0\left(\frac{s}{\omega_c}\right)^2 + a_1\left(\frac{s}{\omega_c}\right) + a_2}{a_2\left(\frac{s}{\omega_c}\right)^2 + a_1\left(\frac{s}{\omega_c}\right) + a_0} \quad (3)$$

where ω_c is the center frequency and a_0, a_1, a_2 are real constants defined as follows,

$$\begin{aligned} a_0 &= \alpha^\alpha + 3\alpha + 2 \\ a_1 &= \alpha^\alpha - 3\alpha + 2 \\ a_2 &= 6\alpha \tan \frac{(2-\alpha)\pi}{4} \end{aligned} \quad (4)$$

By setting $\omega = \omega_c = 1$ and ensuring $a_0 > a_2 > 0$, $\arg\{T(j1)\} > 0$, thus approximation (3) behaves as a fractional-order differentiator around ω_c . The phase flatness of the approximation frequency response is guaranteed if $\arg\left\{T\left(\frac{s}{\omega_c}\right)\Big|_{j\omega_c}\right\} = \pm\alpha\pi/2$.

In the next section, synthesis of fractional-order PID controller, simulation and experimental results will be described.

Results

In the present section, the structure of the non-integer approximation of a PID controller to regulate voltage in a Buck converter is synthesized. The resulting controller is tested in a classical closed-loop diagram with unit feedback to determine its efficiency through numerical simulations. Experimental results are provided in this section as well.

Controller design

The fractional-order structure of a PID controller was described as follows (Podlubny et al., 2002),

$$G_c(s) = k_p \left(1 + \frac{1}{T_i s^\alpha} + T_d s^\mu\right) \quad (5)$$

where k_p, T_i, T_d are the proportional gain, integral and derivative time constants, respectively, and $0 < \alpha, \mu < 1$. It has been determined through different optimization strategies that derivative and integral time constants must be related as $T_i = \eta T_d$ to obtain a unique solution for (5) (Wallén et al., 2002), thus, by

considering $\alpha = \mu$ and $\eta = 1$ and a slight modification of (5), the fractional-order PID controller can be represented as,

$$G_c(s) = \frac{k_c(T_i s^\alpha + 1)^2}{s^\alpha} \quad (6)$$

where $k_c = k_p/T_i$. It is necessary to recall that controller phase and plant phase are related with phase margin as $\varphi_c = \varphi_{dm} - \pi - \varphi_p$, therefore,

$$\alpha = \frac{\varphi_{dm} - \pi - \varphi_p}{\pi/2} \quad (7)$$

where φ_{dm} is the desired phase margin. Notice from (6) that with the appropriate combination of k_c and T_i , the controller $G_c(s)$ can produce a derivative (integral) effect for $T_i \rightarrow \infty$ ($T_i \rightarrow 0$).

Simulation and Implementation Results

The synthesis of the fractional-order PID controller approximation can be achieved through the following steps:

1. Consider the Buck converter transfer function $G_p(s)$ as the uncontrolled plant.
2. Determine the fractional order α through equation (7).
3. Compute the s^α approximation through equation (3).
4. Determine controller structure $G_c(s)$ through equation (6) as function of k_c and T_i .
5. Compute k_c and T_i values that produce the desired controller phase contribution.
6. Determine performance parameters of the closed-loop response.

For this case, the frequency requirements are gain margin $g_{dm} \geq 15$ dB and phase margin $\varphi_{dm} = 45^\circ$. In the absence of controller, the system has a gain margin $g_p = \infty$ and phase margin $\varphi_m = 22.7^\circ$, thus the phase of plant $G_p(s)$ is $\varphi_m = -157.3^\circ$, which implies that controller phase contribution must be $\varphi_c = 22.3^\circ$ to achieve desired phase margin φ_{dm} , therefore, according to equation (7), the fractional order necessary to control the plant is $\alpha = 0.2481$. Frequency response of approximation s^α and controller $G_c(s)$ corroborate the computations described, see Figure 2.

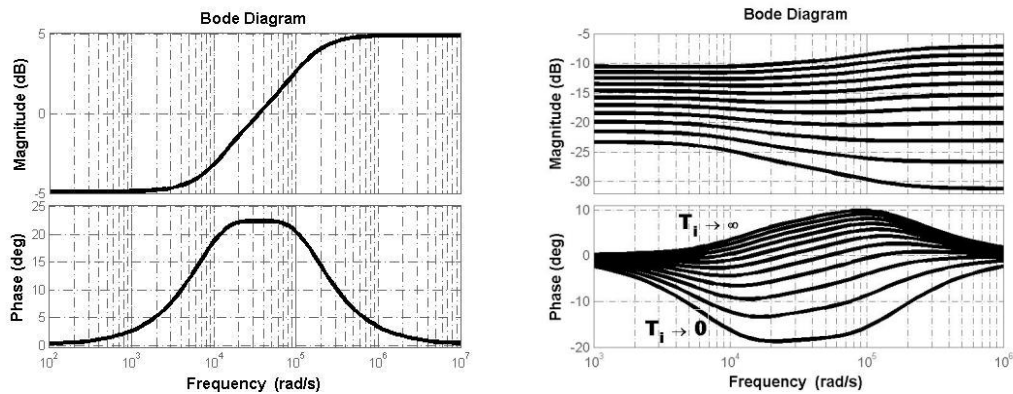


Figure 2. Frequency Response of Approximation s^α (Left) and Controller $G_c(s)$ (Right)

The controller structure is described as follows,

$$G_c(s) = \frac{k(s^4 + \rho_1 s^3 + \rho_2 s^2 + \rho_3 s + \rho_4)}{s^4 + \beta_1 s^3 + \beta_2 s^2 + \beta_3 s + \beta_4} \quad (8)$$

which was obtained with $k_c = 0.035$ and $T_i = 11$. For controller coefficients see Table 2 columns 1 to 4.

Table 2. Controller Coefficients and Constants for Electrical Arrangement

Parameter	Value	Parameter	Value	Constant	Coefficient
ρ_1	1.617×10^5	β_1	2.1507×10^5	$A_1 = -3.476$	$R_1 C_1 = 8.5 \times 10^{-6}$
ρ_2	8.1389×10^9	β_1	1.3653×10^{10}	$A_2 = 0.0129$	$R_1 C_2 = 15 \times 10^{-6}$
ρ_3	1.292×10^{14}	β_1	2.7389×10^{14}	$A_3 = -1.5618$	$R_1 C_3 = 53 \times 10^{-6}$
ρ_4	6.3798×10^{17}	β_1	1.6217×10^{18}	$A_4 = 0.0287$	$R_1 C_4 = 93 \times 10^{-6}$
		k	8.2363	$A_5 = 8.2363$	

To show the effectiveness of the controller, the closed-loop step response is analyzed through performance parameters and thus determine quantitatively the system tracking and regulation capability. On the other side, frequency response is also obtained to corroborate that gain and phase margins are achieved, see Figure 3.

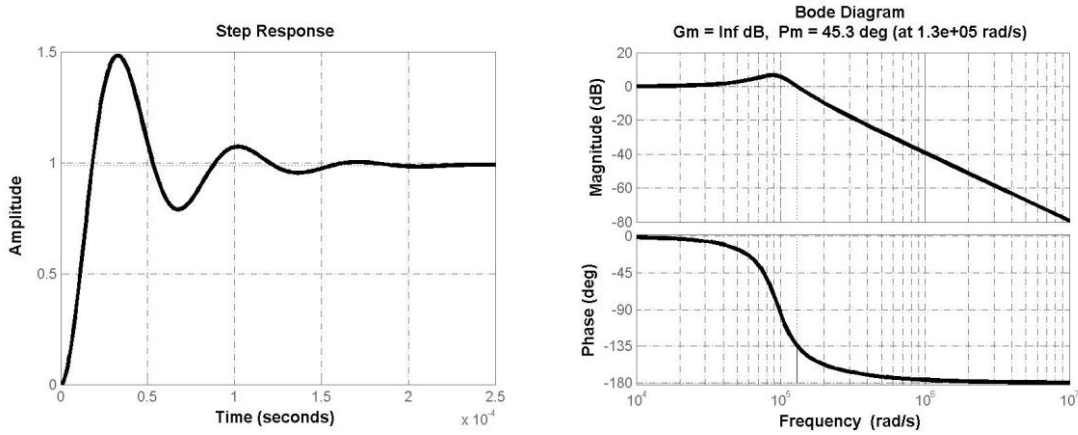


Figure 3. Closed-Loop Step Response (Left) and Frequency Response (Right)

By using the MatLab preloaded tuning algorithm, which considers both performance and stability and targets a phase margin of 60°, a typical PID controller was synthesized. Despite the good operation of the integer-order controller, the fraction-order PID approximation outperforms its regulation velocity, as can be seen in Table 3. Note that the proposed approach produces smaller time constants compared to those generated by the classic one. Even when

Table 3. Performance Parameters of Fractional-Order PID vs typical PID

Parameter	Notation	FO PID App	Typical PID
Steady-state error	e_{ss}	0.0001	0
Time constant	τ	13.1 μ s	148.01 μ s
Rising time	t_r	16.7 μ s	191.5 μ s
Peak time	t_p	32.5 μ s	358 μ s
Settling time	t_s	0.147 ms	1.47 ms
Overshoot	%M	50 %	39 %
Phase margin	Pm	45°	45°

Experimental validation will allow us to corroborate superiority of fractional-order PID approximation and viability to be used in highly efficient converters such as Silicon-Carbide ones. Prior to synthesize the electrical diagram of the proposed controller, the partial fractional expansion of (8) will be given as follows,

$$G_c(s) = A_1 \left(\frac{1}{R_1 C_1 s + 1} \right) + A_2 \left(\frac{1}{R_1 C_2 s + 1} \right) + A_3 \left(\frac{1}{R_1 C_3 s + 1} \right) + A_4 \left(\frac{1}{R_1 C_4 s + 1} \right) + A_5 \quad (9)$$

with constant values listed in Table 2 columns 5 and 6. Note that factors accompanying coefficients A's resemble an RC circuit transfer function, thus, the electrical arrangement for the fractional-order PID approximation will be implemented through RC circuits and OPAMPs in adder configuration, see Figure 4 (left).

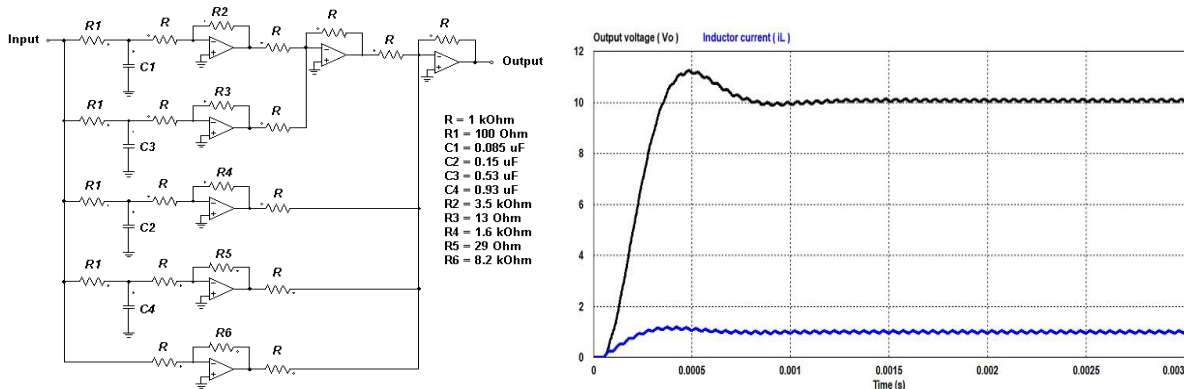


Figure 4. Controller Electrical Circuit (Left) and Output Voltage/Inductor Current (Right)

Electrical simulations of the synthesized fractional-order PID approximation allowed us to corroborate effectiveness of the proposed approach. The controller successfully regulated voltage in the Buck converter while operating in continuous conduction mode, see Figure 4 (right).

Experimental results confirmed that the proposed controller is realizable due to simplicity and commercial parameter values. The electrical arrangement of the controller successfully achieved output voltage (purple) regulation in a Buck converter, see Figure 5. In addition to the effective regulation, the controller exhibited good tracking characteristic, see Figure 6.

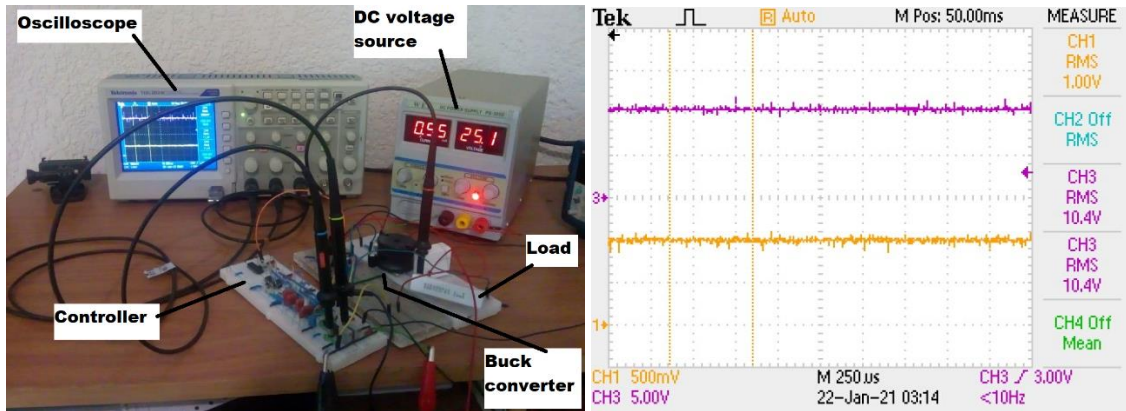


Figure 5. Experiment Table (Left) and Confirmed Regulation of Output Voltage (Right)

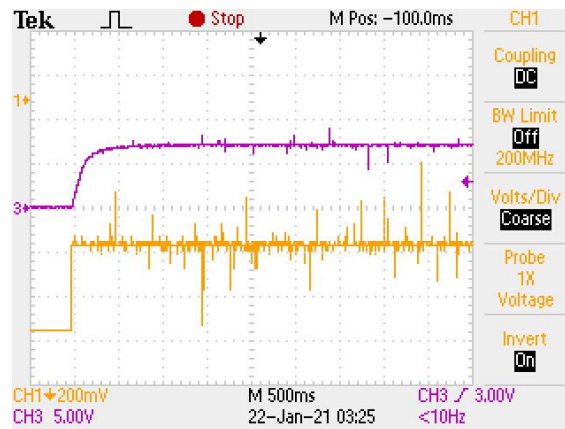


Figure 6. Confirmed Tracking Regulation Capacity of Controller

It is worth mentioning from Figure 6 the regulation velocity. Note that the output voltage (purple) can follow the reference value (yellow) in a very short amount of time. This confirms what it was believed on the viability of the proposed approach. Therefore, authors concluded that the suggested controller effectively regulates voltage in a Buck converter, and it is an alternative to be used in highly efficient Silicon-Carbide converters.

Discussion

The paper addressed the effectiveness of a fractional-order PID controller approximation. The proposed method considers both desired closed-loop characteristics and robustness. The synthesis of the controller is achieved through a biquadratic module that exhibits flat phase response. Fractional calculus was considered due to it has proven to be efficient describing systems with higher accuracy and robustness against parameter variations/uncertainties.

Viability of the proposed approach was investigated to determine if it represents an alternative to be used in highly efficient converters such as Silicon-Carbide ones. Experimental results confirmed effectiveness of the controller regulating output voltage in a Buck converter through a single control loop. These results open the possibility of applying this approach to a current control mode and thus determine if regulation velocity can be enhanced even more.

Conclusion

In this paper, the viability of a fractional-order PID controller approximation to regulate voltage in a Buck converter was investigated. The non-integer approach is integrated in the controller synthesis through biquadratic modules that exhibit flat phase response. The approximation of fractional-order Laplacian operator was used directly in the standard structure of a PID controller. The tuning process was reduced to determine the controller required effect through time constant T_i . The resulting controller transfer function is represented into its partial fraction expansion, which is used to generate the electrical arrangement for the implementation.

The resulting controller effectively regulated voltage of a Buck converter in a classical control diagram with unit feedback. Performance parameters allowed us to determine that the controller produced a fast response with stable regulation and tracking characteristic. By comparing time constants with those obtained with a typical PID controller, we determined quantitatively the superiority of the suggested approach. Experimental results are the confirmation that the proposed methodology works, therefore, fractional-order PID approximation can be a viable alternative to be used in more efficient converters.

Recommendations

As future work, the development of the equivalent current mode control under this method is suggested. Since the regulation velocity and a better tracking characteristic were the improvements, it is expected that a control strategy that considers a more dynamic variable as is the current will derive in the enhancement of system time constants.

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References

- Soriano-Sánchez, A. G., Rodríguez-Licea, M. A., Pérez-Pinal, F. J., & Vázquez-López, J. A. (2020). Fractional-order approximation and synthesis of a PID controller for a buck converter. *Energies*, 13(3), 629.
- Sitbon, M., Aharon, I., Averbukh, M., Baimel, D., & Sassonker, M. (2020). Disturbance observer based robust voltage control of photovoltaic generator interfaced by current mode buck converter. *Energy Conversion and Management*, 209, 112622.
- Alim, M. W., Windarko, N. A., & Rakhmawati, R. (2020). Fuzzy Logic Control Design on Buck Converter for Thermo Electric Air Cooler Power Supply. *JAREE (Journal on Advanced Research in Electrical Engineering)*, 4(2).
- Zhang, L., Wang, Z., Li, S., Ding, S., & Du, H. (2020). Universal finite-time observer based second-order sliding mode control for DC-DC buck converters with only output voltage measurement. *Journal of the Franklin Institute*, 357(16), 11863-11879.
- Hernández-Guzmán, V. M., Silva-Ortigoza, R., & Orrante-Sakanassi, J. (2020). Velocity Control of a PMSM Fed by an Inverter-DC/DC Buck Power Electronic Converter. *IEEE Access*, 8, 69448-69460.
- Junior, F. A. D. C. A., Bessa, I., Pereira, V. M. B., da Silva Farias, N. J., de Menezes, A. R., de Medeiros, R. L., ... & da Costa Júnior, C. T. (2020). Fractional Order Pole Placement for a buck converter based on commensurable transfer function. *ISA transactions*, 107, 370-384.
- Hekimoğlu, B., & Ekinci, S. (2020). Optimally designed PID controller for DC-DC buck converter via hybrid whale optimization algorithm with simulated annealing. *Electrica*, 20(1), 19-28.
- Nishat, M. M., Faisal, F., Evan, A. J., Rahaman, M. M., Sifat, M. S., & Rabbi, H. F. (2020). Development of Genetic Algorithm (GA) Based Optimized PID Controller for Stability Analysis of DC-DC Buck Converter. *Journal of Power and Energy Engineering*, 8(09), 8.
- Saleem, O., Awan, F. G., Mahmood- ul- Hasan, K., & Ahmad, M. (2020). Self- adaptive fractional- order LQ- PID voltage controller for robust disturbance compensation in DC- DC buck converters. *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields*, 33(4), e2718.
- Valele, W., Virambath, R., Mehta, U., & Azid, S. (2020). Fractional analog scheme for efficient stabilization of a synchronous buck converter. *Journal of Electrical Engineering*, 71(2), 116-121.
- Jain, S., Hote, Y. V., Dehuri, P., Mittal, D., & Siddhartha, V. (2020). Experimental validation of fractional order

- internal model controller design on buck and boost converter. *Measurement and Control*, 0020294020922264.
- Abdelmalek, S., Dali, A., Bakdi, A., & Bettayeb, M. (2020). Design and experimental implementation of a new robust observer-based nonlinear controller for DC-DC buck converters. *Energy*, 213, 118816.
- Delavari, H., & Naderian, S. (2019). Backstepping fractional sliding mode voltage control of an islanded microgrid. *IET Generation, Transmission & Distribution*, 13(12), 2464-2473.
- Saleem, O., Shami, U. T., & Mahmood- ul- Hasan, K. (2019). Time- optimal control of DC- DC buck converter using single- input fuzzy augmented fractional- order PI controller. *International Transactions on Electrical Energy Systems*, 29(10), e12064.
- Farsizadeh, H., Gheisarnejad, M., Mosayebi, M., Rafiei, M., & Khooban, M. H. (2019). An intelligent and fast controller for DC/DC converter feeding CPL in a DC microgrid. *Ieee Transactions on Circuits and Systems II: Express Briefs*, 67(6), 1104-1108.
- El-Khazali, R. (2013). Fractional-order $PI^{\lambda}D^{\mu}$ controller design. *Computers & Mathematics with Applications*, 66(5), 639-646.
- El-Khazali, R. (2015). On the biquadratic approximation of fractional-order Laplacian operators. *Analog Integrated Circuits and Signal Processing*, 82(3), 503-517.
- Podlubny, I., Petráš, I., Vinagre, B. M., O'Leary, P., & Dorčák, L. (2002). Analogue realizations of fractional-order controllers. *Nonlinear dynamics*, 29(1), 281-296.
- Wallén, A., ÅRström, K. J., & Hägglun, T. (2002). Loop- Shaping Design of PID Controllers with Constant T_i/T_d RATIO. *Asian Journal of Control*, 4(4), 403-409.