

## A Modified Method for Beam-Forming Using Covariance Matrix in MIMO Radar System

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**Abstract:** A MIMO radar provides solution for beamforming, since transmit beampattern synthesis is well proven method for stimulating the antenna array to develop the beampattern. It is close to the desired one which can give minimum error between these two. A covariance matrix in beamforming contains the direction of invariable data and magnitude distribution in multidimensional space which decides the closeness with the desired beampattern. Here, selection of an optimal covariance matrix is constrained optimization problem with minimization of cost function. In high directivity radar systems, the antenna beam is needed to be steered to cover a large area for detection. In this proposed work, minimization of convex function and transmit beampattern synthesis is carried out with modifications of the covariance matrix ' $R_{cov}$ ' for minimum and maximum transmission power utilization constraints. The beampattern design problem is reformulated here, as semi-definite programming optimization problem and solved using convex optimization, with MATLAB simulation platform. The results show that the beampattern generated using this modified covariance matrix is optimally close to desired pattern.

**Keywords:** Beampattern synthesis, Covariance Matrix, Convex Optimization, MIMO radar

### Introduction

#### MIMO Array Radar Processing

The phased array radar is upgraded by digital receivers in which the waveform generators are distributed across the aperture. This is basic structure of MIMO radar. Closely located antennas, high bandwidth and waveform diversity provides better spatial as well as Doppler resolution as compared to the multistatic radar. Digital signal processing tools helps to estimate the phase difference between two signals. The precise improvement in accuracy of angle estimation and enhancement in angle resolution are observed when the number of array element increases. Thus the promising and cost effective solution to improve the angle resolution is shown by MIMO radar. In this system, orthogonal waveform gives the broad spatial beam pattern for single transmit / receive antenna array. When the spatial signature of the signal lies somewhere within a known linear subspace, multi-rank generalization of the beam pattern is possible to design. Design of the transmit beam pattern requires the waveforms to have arbitrary auto and cross-correlation properties. Design process of beam pattern exists with two approaches. Initially, all the received signals are processed using matched filters and channel properties for each of transmit-receive waveform pair are determined. For this, the signal processing algorithms like constant modulus algorithm are used. But still selection of recovering signal sequences from covariance matrix is a challenging problem. Then in next approach, the focus is on the maximum utilization of gain and energy towards the main lobe and nowhere else. These problems are reformulated as either beam pattern matching design problem or the optimization problems with cost / error functions. In future, flexibility in selection of the signal waveforms and post processing capabilities for improved throughput are the possible options for the advancement in radar systems.

There are many signal successions exists between precise coherence and mutual orthogonality. It is now well proven in the research literature that by selecting the correlation matrix of signal, highly directional spatial beam pattern is feasible to obtain. A covariance matrix in beamforming contains the direction of invariable data and magnitude distribution in multidimensional space. These factors decide the closeness with the desired beam pattern. Here, design of an optimal covariance matrix is presented as constrained optimization problem with minimization of cost function. Direct digital beam waveform synthesis helps in full-fledged use signal transmission power at each time instant and gives more degree of freedom. But this also increases the algorithmic complexity. So these problems are reformulated and by applying the appropriate method or suitable algorithm, the solutions are obtained with lesser complexity.

## Related Work

The performance of radar system is depending on the range, angle and energy reflection from the target. Power aperture product is key measure for the same. In conventional radar arrays, accumulation of phase shifts done coherently to achieve the directionality and same waveform is transmitted. So focused beampattern is achieved by using of large number of array elements. In MIMO radar arrays, wide beampattern is obtained as it transmits the orthogonal waveforms. Hence to get the desired beampattern which will cover the maximum area without wasting the provided power, the necessity of intermediate beamforming design is identified. As a one of the solution of above mentioned problem, the transmit beampattern synthesis work is carried out in literature. This work has two research directions. In first direction, reducing the side lobe level can become an objective and second direction is to design a beampattern which will match the desired beampattern in the considered n-dimensional space [1] [2]. Correlation matrix 'R<sub>cov</sub>' has impactful significance in beampattern design. It can be designed to approximate the resultant beampattern to match with the desired beampattern. So design or modifications in 'R<sub>cov</sub>' will lead to formulate the constrained optimization problems in standard form so providing solution becomes easy. The approach of these optimization is to minimize the cost or error function. This transmit beampattern synthesis is essential for signal strength as well as power estimation. So the minimization of power utilization or minimization of signal error function is formulated as an optimization problem with constraints. Convex optimization techniques can provide the optimal solution to the above mentioned problems [2]. Algorithm complexity of the synthesis problem increases as the degree of freedom increases. Designing the constant modulus signal with fixed cross-correlation is identified as challenging problem and this also possible to address using convex optimization methods. Constant modulus signal is used in many radar systems. These are fully coherent signals, transmitted with full available power. FIR filter design is one of the method to shape the desired beampattern and minimizing the side lobe level. Beam broadening is also possible using the filter design but at the cost of decrement in total power transmit. Now in beampattern synthesis, total power utilization is possible by manipulating the signal cross-correlation [3] [4].

There are number of attempts done to reformulate these problems so that the beamformers can maximize the power in the desired direction and to nullify the power provided to the side lobe level. Separation of spatial and temporal part of the design is one of them which elaborates the close connection between the signal cross correlation properties and the beamformer functioning [4]. In most of the attempts, the synthesis is done by shape approximation which is followed by minimization of MSE between desired and designed beampattern. It requires complex calculations and compromises the efficiency when iterative algorithms are involved. According to the considered spaces like radian, steradian or spherical area spaces, the design problems are reformulated to maintain the accuracy and precise approximation [5] [6]. To reduce this computational complexity, the orthogonal waveforms are pre-processed which results in forming the multi rank beamformers contains the complex weights. But this approach is doubtful as it does not ensure the expected power transmission from each antenna. So independent BPSK or QPSK type of waveforms are preferred and coordinates of hyper-sphere parameterizes the complex weights so equal power transmission is ensured [7] [8]. Convex optimization methods are extensively used to solve these optimization problems as it maintains the specified accuracy and also works effectively within the polynomial dimensions of the problem. In this paper, the total power utilization problem is formulated as semidefinite programming optimization problem and the solution is obtained using convex optimization. Then the covariance matrix is modified to address the power utilization in beampattern matching design. The solution is observed for total transmit power variation.

## System Model of MIMO Array and Processing

Consider for an equal number of transmitting ( $M_t$ ) and receiving ( $M_r$ ) elements in a phased array with  $\lambda/2$  spacing in between them. A narrow band signal  $S_{(t)}$  is transmitted by each element in the direction  $\theta$ . So the output signal of the transmitter by far field approximation, is given as,

$$x_{(t)} = a_{(\theta)} S_{(t)}. \quad (1)$$

$$\text{where, } a_{(\theta)} = \begin{bmatrix} 1 \\ e^{j2\pi d \sin \theta} \\ \vdots \\ e^{j2\pi d (M_t - 1) \sin \theta} \end{bmatrix} \text{ i.e. steering vector and '}\lambda\text{' is the carrier wavelength}$$

Now, lets consider a MIMO radar with M collocated, narrowband transmit/receive antenna elements. These antenna elements form a uniform linear array (ULA) with  $\lambda/2$  spacing between them. Each transmitted signal pulse has N samples. Here, the transmit/receive antennas are assumed to lie along the z-axis. Such structure is shown in [12]. The  $i^{\text{th}}$  element of the array is driven by a signal  $x(n)$ . For non-dispersive propagation, the baseband signal at the target location having target location  $\theta$  is given as,

$$\sum_{i=1}^M e^{-j2\pi f_0 \tau_i(\theta)} x_i(n) = a^H(\theta)x(n) \quad n = 1, \dots, N \quad (2)$$

Here  $f_0$  is the carrier frequency of the radar and  $\tau(\theta)$  is the time taken by a signal emitted from  $i^{th}$  element to reach the target. The array steering vector  $a(\theta)$  is given as,

$$a(\theta) = [1 \ e^{j\pi \sin(\theta)} \ e^{j2\pi \sin(\theta)} \ \dots \ e^{(N-1)\pi \sin(\theta)}]$$

We can write the probing signal power at location  $\theta$  as,

$$P(\theta) = E \{ |a^H(\theta)x(n)|^2 \} = a^H(\theta) R_{cov} a(\theta) \quad (3)$$

where,  $R_{cov}$  is the signal covariance matrix given by the expression:

$$R_{cov} = E \{ x(n) x^H(n) \} \quad (4)$$

The signal power pattern in Eq. 4 as a function of  $\theta$  is the transmit beam pattern that need to be synthesized the covariance matrix  $R_{cov}$  for the transmitted signal, it is possible to

1. Allot the total spatial power in the direction of known target and shrink it elsewhere.
2. Match a desired beam pattern for variable power allotment.
3. Achieve a predetermined 3 dB main beam width and minimize the side lobe levels.

Now let  $B_p(\theta)$  be the desired beampattern and total transmit power constraint are considered for problem design. So, by using two-norm as cost function, the problem can be designed mathematically as:

$$\min_{\alpha, R_{cov}} \sum_{k=1}^K (|\alpha B_p(\theta_k) - a^H(\theta_k) R_{cov} a(\theta_k)|^2) \quad (6)$$

For the constraints,  $\text{tr}(R_{cov}) \leq c$  where 'c' is total available transmission power and  $R_{cov} \geq 0$ . This problem is then reformulated as unconstrained semidefinite programming optimization problem as shown in Eq. 7.

$$\min_{\alpha, R_{cov}} \sum_{k=1}^K (|\alpha B_p(\theta_k) - \hat{a}^H(\theta_k) \hat{R}_{cov} \hat{a}(\theta_k) + d|^2) \quad (7)$$

Where,

$$d = c - \text{tr}(R_{cov}), \quad \hat{R}_{cov} = \begin{bmatrix} R_{cov} & 0 \\ 0 & d \end{bmatrix} \quad \text{and} \quad \hat{a}(\theta_k) = [a(\theta_k) \ \dots \ 1]^T$$

The beampattern synthesis is a problem of designing the suitable signal covariance matrix, such that the obtained pattern matches the desired one. The aim is to minimise a cost function. Eq. 7 is a problem of optimization and is in the form of unconstrained SDP problem. This problem is solved in simulation using MATLAB platform and CVX toolbox.

In this simulation, collocated MIMO radar is considered with 10 array elements separated by  $\lambda/2$  distances.

For  $c=1$  and region  $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ , the desired beampattern given as,

$$B_p(\theta) = \begin{cases} 2, & \theta \in \{\theta'_k - w_b, \theta'_k + w_b\} \\ 0 & \text{Else} \end{cases} \quad (8)$$

Where  $\theta'_k$ = Location of target in expected region and  $w_b$ = half of beam width for each target location.

The simulation results are provided in [13] as a solution of the above mentioned optimization problem. From the obtained response it is observed that, the synthesized beampattern follows the desired pattern. This indicates that the maximum power is utilized in the desired direction. One of the major concern about antenna performance and efficiency is power management. To improve the efficiency, the ohmic losses in antenna must be minimized to reduce power utilization in form of heat loss and it is possible at hardware level. In addition to this, directive signal transmission also reduces the power utilization in unwanted directions. But requirement of variable power allotment with respect to the distance from the detected target is still unaddressed. So reducing the power utilization according to the search range will not only save the power but also show remarkable reduction in system cost. To address this problem, the covariance matrix in Eq. 7 is further modified and a solution is proposed to allot the power as per the search range requirements. This will improve the efficiency and save the system cost with minimum utilization of power.

### A Proposed Solution:

The diagonal values of the covariance matrix  $R_{cov}$  are transmit powers. It should be non-negative and their sum should be equal to maximum available power. This covariance matrix  $R_{cov}$  is modified by considering the factors related to the defined variable d. let's redefine two variables as  $d_1$  and  $d_2$ .

Where,  $d_1 = c - \text{tr}(R_{cov})$ , which states that how much power transmission is not done. And new variable  $d_2$  is defined such as,

$$d_2 = \text{tr}(R_{cov}) - x, \text{ where } 0 < x < 1 \quad (9)$$

This new variable  $d_2$  states that the total power transmission surely done against the considered value of x. Here x indicates desired value of allotted power. So the updated constraints for the optimization problem can be given as,

Total power utilized for any transmission is  $0 < \text{tr}(R_{cov}) < c$  and guaranteed power transmission is  $x \leq \text{tr}(R_{cov}) < c$ .

for the inequality of  $\text{tr}(R_{cov}) \leq c$  to hold,  $d_1$  and  $d_2$  must be non-negative. Hence for,

$$\text{tr}(R_{cov}) < c, d_1 = c - \text{tr}(R_{cov}) > 0$$

and for

$$\text{tr}(R_{cov}) \geq x, d_2 = \text{tr}(R_{cov}) - x > 0.$$

The Covariance Matrix  $R_{cov}$  is updated with  $d_1$  and  $d_2$  variables.

So new modified covariance matrix  $\tilde{R}_{cov}$  is presented as,

$$\tilde{R}_{cov} = \begin{bmatrix} R_{cov} & 0 & 0 \\ 0 & d_1 & 0 \\ 0 & 0 & d_2 \end{bmatrix}$$

Since the sum of Eigen values of a matrix equals to the trace of the matrix so  $\tilde{R}_{cov}$  will be positive only for  $d_1$  and  $d_2 > 0$ . And  $R_{cov}$  positive semidefinite. Thus the Eq. 7 is updated as,

$$\min_{\alpha, \tilde{R}_{cov}} \sum_{k=1}^K (|\alpha B_p(\theta_k) - \hat{a}^H(\theta_k) \tilde{R}_{cov} \hat{a}(\theta_k) + d_1 + d_2|^2) \quad \text{s.t } \tilde{R}_{cov} \geq 0 \quad (10)$$

Now this SDP problem is solved using CVX toolbox in MATLAB simulation environment. An Iterative Algorithm steps for Proposed Solution are as mentioned below.

**Algorithm:**

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Initialize the system parameters
Define the range of angle of interest ( $\theta_k$ ), steering vector ( $a(\theta)$ ) and desired beampattern ( $B_p(\theta)$ ).
Generate the set of steering vector for each angle ( $-90^\circ : 90^\circ$  range).
For K= 1:10,
    Begin CVX with SDP
    Solve the Eq. (10) for optimization considering  $d_1$  and  $d_2$  constraints.
    Update the  $\tilde{R}_{cov}$  for each K
End For
Post process results and visualization
Stop and Exit
    
```

**Results**

In this section, simulation results are presented to show the effectiveness of the modification in covariance matrix. Following parameters are considered for simulation.

Total no. of ULA elements: 10

Element spacing:  $\frac{\lambda}{2}$

Table 1 shows the average values of desired power, obtained power and mean square error with respect to the considered allotted power x. The optimal values of optimization problem given in Eq. 11 are also mentioned in the table. Respective simulation plots of Obtained power vs angle  $\theta$  are shown in Fig. 3(a) to Fig. 3(f).

**Observation Table of Values Obtained for Different Values of x:**

Table 1. Statistics of desired and obtained power values for different values of x

Sr.	Alloted Power X (dB)	Average Desired Power (dB)	Average Obtained power (dB)	Average MSE	Optimal Value (dB)
1	0.2	0.1348	0.1347	8.0000e-06	+0.51026
2	0.4	0.2696	0.2693	3.2000e-05	+1.02052
3	0.5	0.3370	0.3367	5.0000e-05	+1.27564
4	0.6	0.4044	0.4040	7.2000e-05	+1.53077
5	0.8	0.5392	0.5387	1.2800e-04	+2.04103
6	1	0.6740	0.6734	2.0000e-04	+2.55129

Simulation Plots for different values of x:

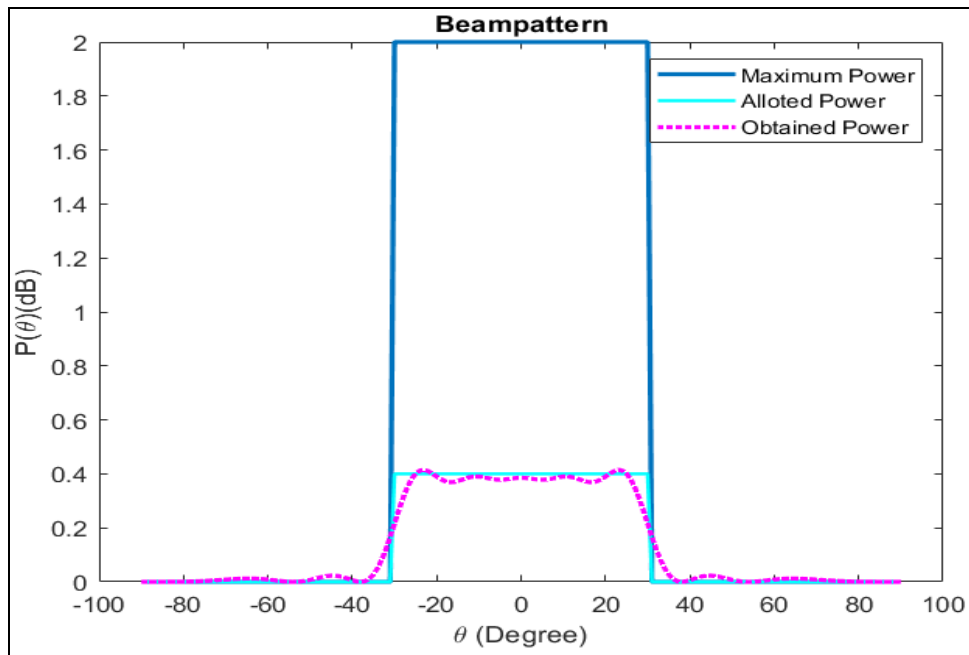


Figure 3(a). Obtained Beampattern for  $x = 0.2$

For  $x = 0.2$ , the average desired power is 0.1348 dB and average power obtained is 0.1347 dB. Thus the average MSE is give by  $8.0000e-06$ . It shows the maximum closeness to the desired pattern.

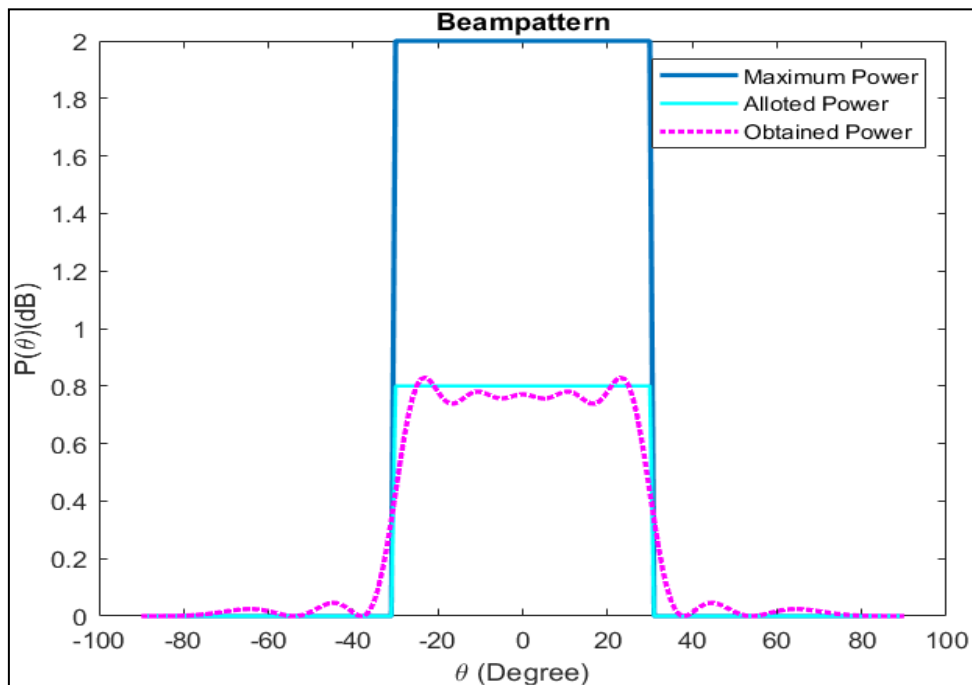


Figure 3(b). Obtained Beampattern for  $x = 0.4$

In Fig. 3(b), the average desired power and average obtained power is 0.2696 dB and 0.2693 dB respectively. The MSE value is  $3.2000e-05$ .

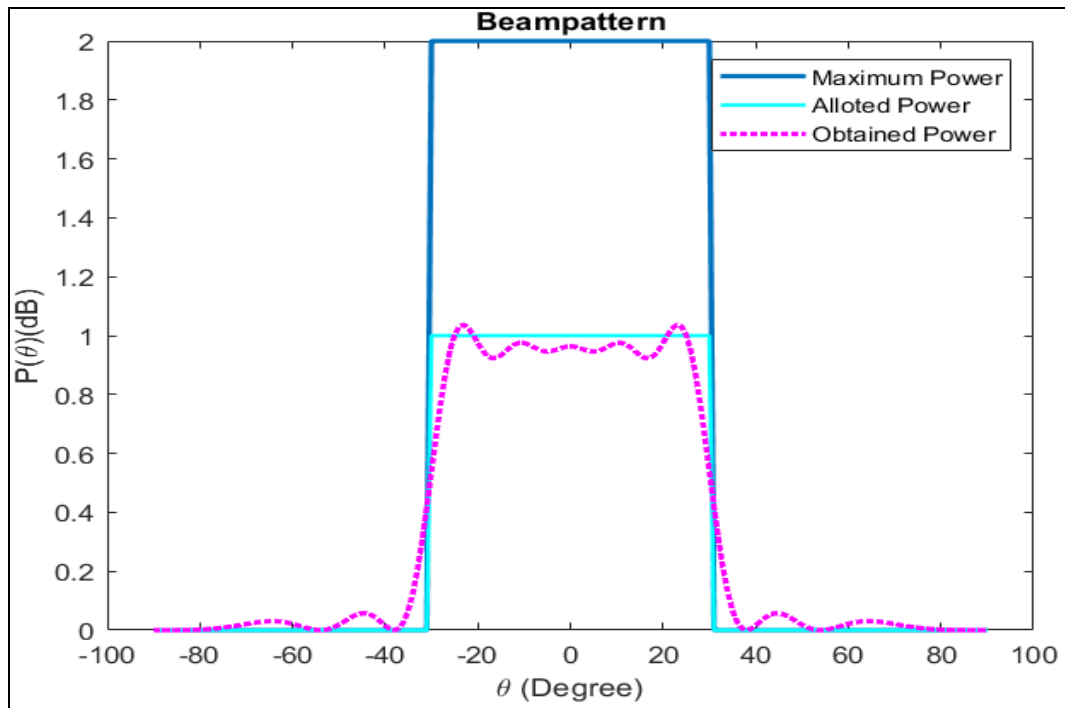


Figure 3(c). Obtained Beampattern for  $x = 0.5$

For half power allocation i.e. when  $x = 0.5$ , the average power obtained is 0.3367 dB against the desired power 0.3370 dB. And  $5.0000e-05$  is the MSE between these two.

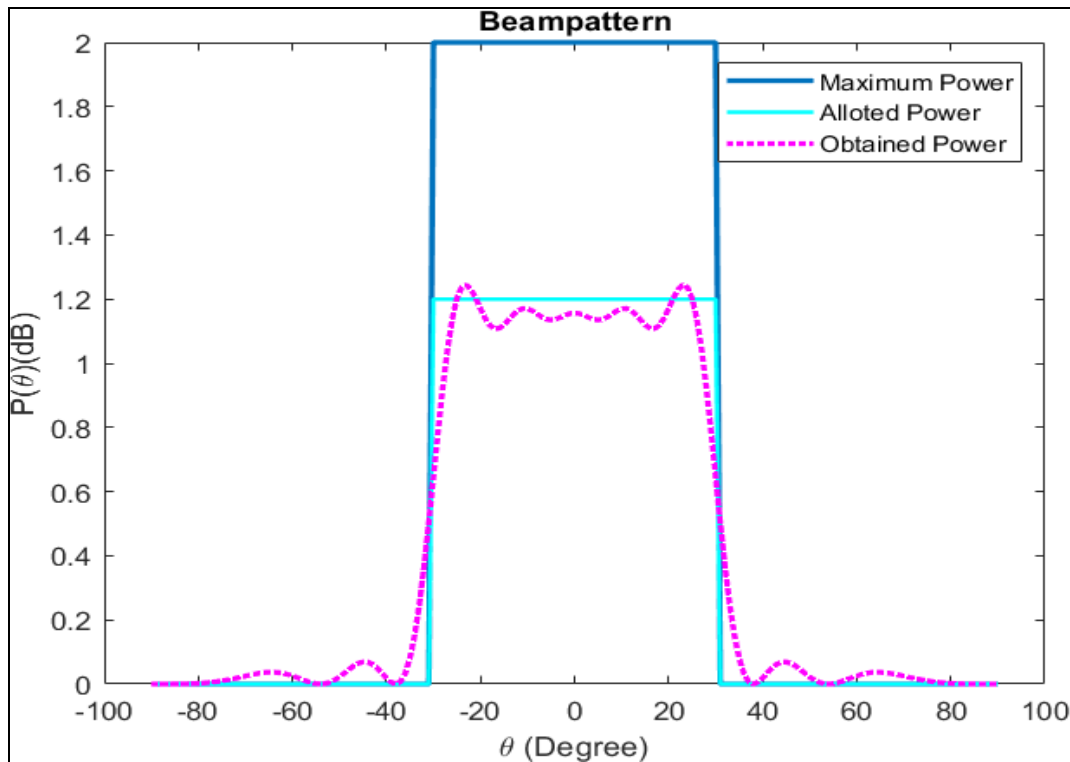


Figure 3(d). Obtained Beampattern for  $x = 0.6$

As  $x$  is increased to 0.6, the Obtained MSE is  $7.2000e-05$  for the value of desired power 0.4044 dB and the obtained power 0.4040 dB.

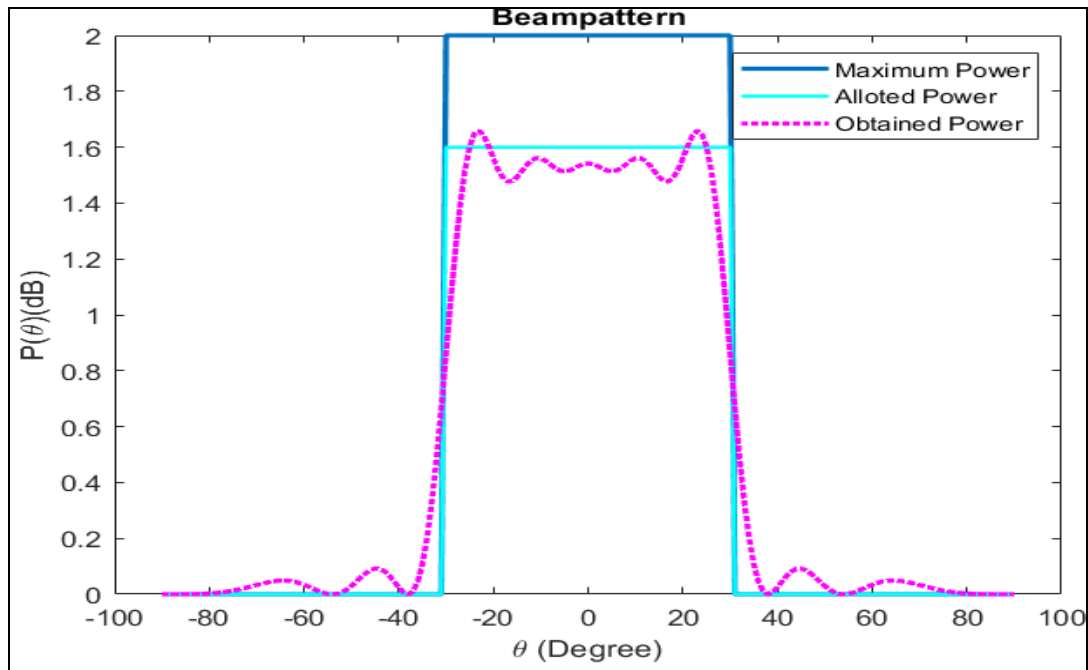


Figure 3(e). Obtained Beampattern for  $x = 0.8$

For  $x=0.8$ , the average desired power is 0.5392 dB and obtained power is 0.5387 dB. So the  $1.2800e-04$  MSE is observed.

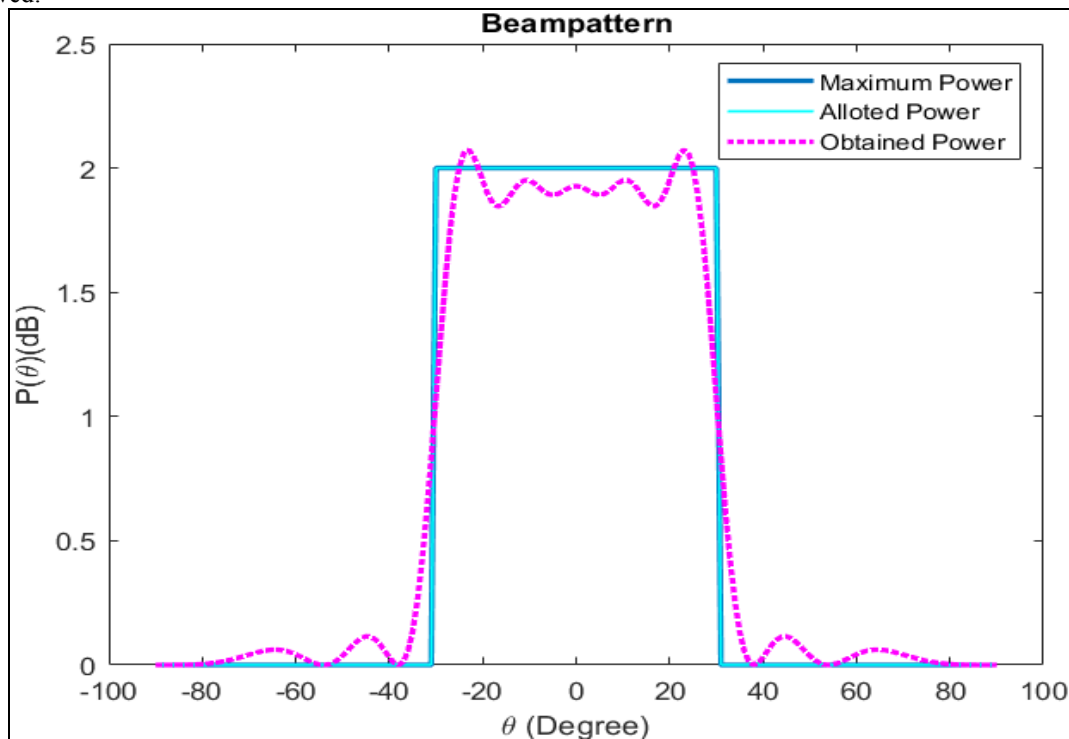


Figure 3(f). Obtained Beampattern for  $x = 1$

When  $x = 1$  the maximum available power is utilized by the Array and the 0.6734 dB power is obtained. The desired average power is 0.6740 dB. So the MSE observed is  $2.0000e-04$ .

In this simulation, it is shown that using beampattern synthesis, approximation of the beampattern in the desired direction is possible. By designing the covariance matrix, SDP optimization problem is formulated to optimize the total power transmission. Further modification in Covariance matrix is done to vary the allotted power with respect to power transmission constraints. From Table 1, it is observed that maximum power is utilized in the desired direction. Average MSE values indicated the closeness of obtained power to the total desired power.



Fig.3(a) to Fig. 3(f) shows the beampattern for different power allotment. It is seen that maximum MSE is occurring at the transition from side lobe to main lobe region or main lobe to side lobe transition. Power utilization in side lobe region is below -3dB i.e. less than the half power band width. This indicates the effectiveness of the method.

## Conclusion

Modified method is presented here investigates the beampattern. The modified covariance matrix offers the higher degree of closeness towards the desired beampattern. The semidefinite optimization method is used for optimizing the transmitter power requirements. Proposed method uses the convex optimization covariance method. Simulation tools namely MATLAB and CVX toolbox is used for simulation purpose. Obtained results justifies the objective of the proposed method. It is possible to provide the variable power to the array elements according to the target distance range detected in radar surveillance. This will result in the efficient utilization of total transmitted power hence improves the energy efficiency of the system.

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